В. ЛЁВШИН

ТРИ ДНЯ В КАРЛИКАНИИ

Сказка да не сказка

Three days in Dwarfland

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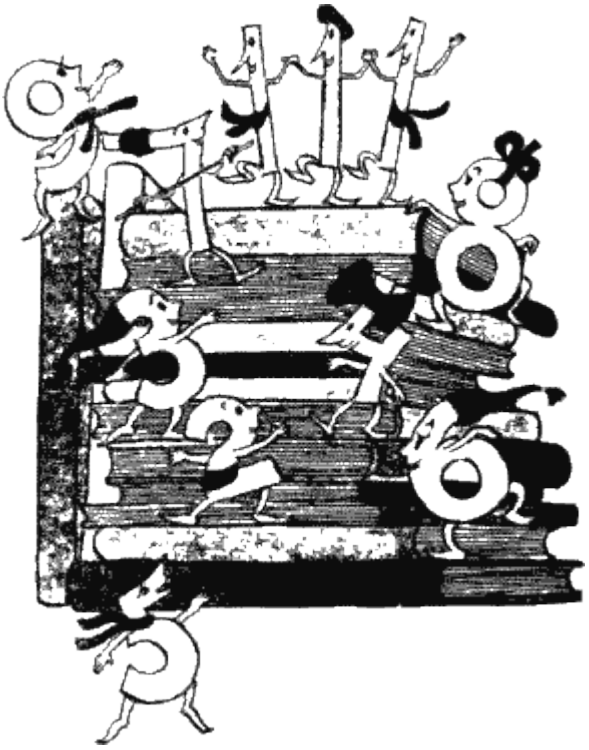

First Day

Let’s Go!

"Who among you has been to Dwarfland?" I asked.

The children exchanged puzzled glances.

"What is Dwarfland?"

"Where is it located?"

"Who lives there?..."

I raised my hand — the children fell silent.

"So none of you has been to Dwarfland?... That’s a pity. Dwarfland is a very interesting country. I’ve traveled across this country far and wide, made friends with all its inhabitants, and regularly correspond with them."

The children listened in amazement.

"Would you like to travel with me to Dwarfland?" I asked.

"Of course we want to!"

"Lead us to Dwarfland!"

"And I will," I answered.

"Right now?"

"We can go now. Just know that the journey won’t be easy."

"All the better," declared Seva. "I’ll pack my backpack in no time: toothbrush — one, towel — two, mug — three... In short, everything like in a hiking trip. Right?"

"No, that’s wrong," I replied. "No soap, no toothbrush. Dwarfland is a very special country. There isn’t even any water there."

"How do they wash themselves?" Tanya spread her hands in confusion. "Do they really walk around dirty?"

"No, why would they," I objected, "the inhabitants of Dwarfland wash themselves... with erasers, ordinary school erasers."

The children laughed.

"We should try that," said Seva.

"And what are these odd people called?"

"Since they live in Dwarfland, they're called dwarves," I answered.

"Well, fine," Tanya persisted, "let's say the dwarves wash with erasers. Fine. But what do they drink if they have no water?"

"Probably coffee or cocoa," remarked Seva.

"As if — cocoa!" objected Tanya. "You can't make cocoa without water."

"I know!" Seva rejoiced. "They drink carrot juice."

"I don't like carrot juice," Tanya grimaced. "Grape juice is tastier. The dwarves drink grape juice."

"No, kids," I intervened, "you'll never guess what the dwarves drink instead of water."

"Ink!" blurted out Seva and got scared of his own wit. Everyone laughed again.

"But you guessed right," I said. "The dwarves indeed drink ink."

"Blue or red?" Seva asked importantly, pleased with his joke.

"Both blue and red," I answered, "and green, and purple. And if there's no ink, the dwarves drink paint."

"How can that be?" Tanya was puzzled. "You can't make ink without water either."

"They get ink delivered to them ready-made," I answered. "From another country."

"From Inkland!" Seva added triumphantly.

"Stop your nonsense!" Tanya stopped him. "They don't like that in Dwarfland."

And so we prepared for the journey.

Three people set out with me: Tanya, Seva, and Oleg.

Oleg, as you've already noticed, didn't say a word the whole time. He's very quiet. But when he does say something, it's always appropriate and always correct. That's why they nicknamed him "Oleg the Seer."

As for Seva — he never closes his mouth, even when he's alone. On the street, he reads signs aloud, talks to dogs he meets, or even to himself, for which Tanya often scolds him. After all, Tanya is the best student in the class, so she acts a little superior.

Arabella

We entered the city unnoticed.

It was an extraordinarily beautiful city. In its center was a large circular square. Nine streets radiated from it like rays.

The streets were named accordingly: "Street 1," "Street 2," and so on up to "Street 9."

And the square itself was called Numerical Square.

The streets were crossed by numerous lanes and alleyways, so that you could always get from one street to another without going through Numerical Square.

The lanes also had their own names: "Fractional," "Decimal," "Common"... There were even some "Periodic" ones — extremely long lanes that extended far beyond the city, somewhere to the edge of the world. Some lanes ended in dead ends. In addition, the city was crossed by wide avenues, alleys... In the center of Numerical Square stood an enormous glass building, on whose high tower shimmered a sign in all the colors of the rainbow:

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ARABELLA, CAPITAL OF DWARFLAND
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We silently emerged onto Street 8, where completely identical eight-story buildings stood. Each building had eight doors, each floor had eight windows. And imagine, all the houses on this street were marked with the same number — 8!

Tanya was the first to break the silence.

"How do postmen deliver letters if all the houses have the same number?"

Seva, glad for a chance to speak, opened his mouth, but just then a song was heard from the window of one of the houses. It was probably a mother singing her child to sleep.

> Sleep, little Zero, sleep tight,
> Dawn is approaching the night,
> Seven hours and seven minutes remain
> Till sunlight streams through the pane.
> Twelve kittens curl up in a heap,
> Thirty mice in their holes sleep deep,
> Even the elephant, tons in weight,
> Dreams his hundredth dream to date.
> Sleep, little Zero, sleep tight,
> Dawn is approaching the night,
> Seven hours and six minutes remain
> Till sunlight streams through the pane.

> To grow strong as the years pass by,
> Never rush forward or try to fly.
> Stay humble, modest, and meek,
> And tenfold strength you will seek.
> Sleep, little Zero, sleep tight,
> Dawn is approaching the night,
> Seven hours and five minutes remain
> Till sunlight streams through the pane.

The singing stopped. There was a light slap, and the woman said:

"Sleep, you rascal! Only seven hours and four and a half minutes remain until the new day. If you don't fall asleep right now, you'll remain a zero all your life. Sleep! What did I tell you?!"

We moved further on tiptoe and turned into a small dead end that ended with a large shed. Seva immediately read the sign:

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Warehouse

DWARVES, ADD EVERYTHING HERE!
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Despite it being night in Arabella, incredible noise was coming from the warehouse. People were shouting, arguing, moving some bulky items.

We came closer and this is what we heard.

"Girl, why are you adding oranges here?!" a deep bass voice was indignant. "Can't you see that electric light bulbs are stored here? Light bulbs should be added with light bulbs, and oranges with oranges. Otherwise, the sum will be some kind of lightranges! What do they even teach you at school? It's obvious that you're a little Two. Yes, yes. A Two, and nothing more! Tomorrow you'll be adding frogs with herons, and nothing will remain of your frogs — the herons will simply gobble them up!"

"But why did you yourself add a white roll with ham?" objected a thin little voice.

"Oh, you ignoramus!" the bass voice was outraged. "I didn't add them — I made a sandwich out of them. That's a completely different matter! A ham sandwich is very tasty! How dare you teach me?! First, live to my age, then teach others. And I'll somehow figure out for myself what to eat my ham with."

"Hee-hee-hee!" the girl laughed. "You're just a glutton!"

"And you're an ignoramus!" the bass voice raged. "Get out of here, or else tomorrow I'll tell everything to your teacher."

Not waiting to meet the arguers, we hurried to get out of the dead end.

"Listen," said Seva, "now I think I understand what Dwarfland is. It’s an Arithmetic state!"

"Oh, well done, Seva!" Tanya quipped. "How did you ever figure it out? Weren’t you yourself adding tangerines with light switches yesterday?"

Seva glanced guiltily at me out of the corner of his eye. But I pretended I hadn’t heard anything.

We returned to Numerical Square. Dawn was breaking. Shutters gradually opened, and the first passersby appeared on the street.

The day had begun in Arabella.

None of the city’s residents had seen us yet. We took shelter in a small garden. And I began to tell my companions about how this state came into being.

The Most Ancient State

We know of many ancient states: India, Egypt, Babylon, Assyria, Greece... We even know approximately when each of them appeared. But nobody knows when the Arithmetic State appeared. We can conclude that it is very, very ancient from the fact that in Babylon, Egypt, Greece, Rus', and all other ancient states, the Arithmetic State is mentioned. This means it is older than all of them.

Perhaps it was founded by the very, very first human on earth, so ancient that there was no one more ancient than him? Perhaps he issued a Decree on the establishment of the Arithmetic State? Or perhaps he captured some country by force and renamed it in his own way?

No, this couldn’t be. The very first human certainly couldn’t write decrees — he couldn’t write at all, and there weren’t any states at that time.

The ancient human had a wife and two children. One day, the very, very first human went hunting and killed the very, very first wild boar. He came home and... what did he do with his prey? Well, of course, he divided it into four parts: for his wife, his son, his daughter, and himself.

This is how the arithmetic operation of division came into the world. That’s how the ancient human laid the first stone of the Arithmetic State!

And then it took off! Children, like all children, wanted to eat. Food had to be stored for the future. The ancient human started to go hunting more often, and was adding his prey to a pit.

Do you see what he was doing? He was doing addition!

And in autumn, he had to collect many nuts and berries — after all, children love treats. The ancient human's household grew and multiplied.

And when the children grew up, they married the children of another ancient human. They needed to set up their own households. Then the parents began, without regret, to subtract from their possessions the best animal skins, the largest nuts, and fruits, and give them to their children. The parents had, let's say, thirty nuts each, and after the wedding, they had only eighteen left. That means they gave away twelve nuts each.

Tell me, please, isn't this the most common operation — subtraction?

But the ancient human didn't yet know what these arithmetic operations were called. He didn't know arithmetic at all.

Of course, this was a very long time ago. We can only guess how it all happened. More and more people appeared on earth, and their households grew. It became increasingly difficult to divide, add, multiply, and subtract.

And some dishonest ancient people took advantage of this.

"Hey, friend!" said one such dishonest ancient human. "You're cheating me. You promised to give me ten boar legs. Yesterday you gave four, today — five, and you say we're even. Where's the other leg?"

"No, friend," replied the honest ancient human, "I gave you not four, but five boar legs yesterday. You forgot."

"No, it's you who forgot!" objected the dishonest human. "You're no longer my friend, and I'll kill you with this club!"

Of course, none of this would have happened if the honest human had recorded how many boar legs he gave to the dishonest human. But he didn't do this. He didn't do it because he didn't know how to record numbers.

So the honest ancient people decided to do this: represent each received or given boar leg with a pebble, and hide the pebbles in a safe place. Now no one could say they had received four legs instead of five.

That's what they started to do. But then they got confused again. It was fine when they had to count boar legs. There weren't that many of them. Try counting nuts or berries this way! How many pebbles would you have to carry around?

"We've got it!" decided some. "We'll do without pebbles. We'll mark each nut or each boar leg with a notch on the wall. We'll make marks and count them."

"What are you thinking!" objected others. "You'll ruin all the walls in the caves. And you'll still lose count. We need to come up with something smarter and simpler."

It's easy to say "simpler"! This is not a simple task! A lot of water flowed under the bridge before people figured out how to solve it, and new unusual beings — digits — appeared in the world.

These digits looked little like the ones you know. But I'll tell you about that later. And now... Since we are in Arabella, let's talk about the digits that live in this city.

These digits were invented in ancient India. And they should have been called Indian. But nobody in those times knew about the Indians' invention. India was conquered by the Arabs, who destroyed cities and took many treasures from there. And along with them, the digits. So we learned about the Indian invention through the Arabs and began to call these digits Arabic.

In ancient times, there were nine such digits: 1, 2, 3, 4, 5, 6, 7, 8, 9. They were the ones who founded this state. And they named its capital Arabella.

Now do you understand where we've ended up?

Look: someone is opening the gate. It seems we'll have to apologize for our uninvited intrusion.

The Apple Orchard

No sooner had I said this than the garden filled with cheerful little ones. They were led by an older number — a neat Four with a ribbon in her hair. After carefully smoothing the folds of her school dress, she approached us and politely greeted us.

"Excuse me," I said, "we entered your orchard without permission."

"In our city, everyone is welcome," she said, "especially schoolchildren."

"How do you know I'm a schoolboy?" asked Seva. Four smiled slyly:

"We've met many times before. I've even had to appear on the pages of your report card. To tell the truth, not as often as I would have liked."

"There are report cards where you've never had the chance to visit at all," Seva retorted, giving Tanya a meaningful look.

"Yes, but that's a completely different matter. My friend Five is a regular guest there. She's much better than me, and I'm not at all upset about it."

Seva blushed and was about to answer something, but at that moment — and quite opportunely — the little ones ran up to Four.

"Oh, what beautiful apples in this orchard! Can we try them?"

"Why not," said Four, "but first you need to pick the apples."

"We wanted to, but we couldn’t. They’re too high."

"Don’t you know our rule? The apples will fall to the ground by themselves — you just need to solve a problem."

To our surprise, the little ones weren’t upset at all. They busily took small sticks out of their pockets and prepared to write the problem conditions in the sand.

"So," Four continued, "apples are lying on three plates. Half of all the apples are on the first plate. When half of what was on the second plate was taken from the first plate, and then half of what was on the third plate was also taken from the first plate, only two apples remained on the first plate. The question is, how many apples were initially on each plate? Understand?"

The little ones concentrated intensely, moving their sticks through the sand; some even stuck out their tongues from the effort. Soon, however, their mood clearly worsened. Many even began to cry. Four wasn’t surprised at all by this, took out a dazzling white handkerchief, and carefully wiped the small wet noses.

"No need to cry," she said, "this problem is still a bit difficult for you. Let our guests solve it. And then we’ll all try these wonderful apples together."

"Tanya, all hope is on you!" Seva whispered. He had been eager to get better acquainted with the apples for a while.

Very soon the orchard was literally covered with ripe fruit.

"Well done, Tanya!" Seva exclaimed in delight. "I knew you would solve the problem!"

The little ones clapped their hands in unison and rushed to pick up the apples. But Tanya stood embarrassed, her cheeks burning.

"I didn’t solve the problem!" she said with difficulty and, covering her face with her hands, turned away.

"Well, I never! Who did then?" the little ones grew excited.

"Not me, in any case!" Seva grumbled.

Then everyone looked at Oleg. As usual, he was silent. And next to him in the sand, everyone saw three numbers. It was the answer to the problem.

"Absolutely correct!" said Four, looking at the numbers, and immediately erased them with her foot.

"Why, why did you do that?" the little ones squeaked.

"Let those who haven’t solved this problem be sure to solve it themselves. And now I must go to the Square of Good Wishes. If you want to see an interesting sight," she kindly addressed us, "I’d be happy to escort you."

We readily agreed and followed our new acquaintance.

Mysterious Signs

The city was teeming with people. With all its streets and numerous lanes, it was like a large but well-studied labyrinth.

This was easy to confirm by seeing how unerringly and quickly the residents of Arabella found their way to the wide Avenue of Active Signs.

Animated dwarves flowed here from everywhere. Among them were children and old people, hurried and unhurried ones, chatterboxes and quiet types, giggly and thoughtful ones. But despite the large crowd, no one pushed anyone, no one stepped on anyone’s feet.

Many nodded to us in friendly greeting, and sometimes even shook our hands — in short, they behaved like good acquaintances.

On both sides of the avenue stretched long buildings with numerous revolving doors. The dwarves kept diving into them and immediately returning with small suitcases in which something melodiously clinked.

At every step, there were signs with large lettering:

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WAREHOUSE OF ACTIVE SIGNS
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Under this inscription was another, smaller one:

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Save on the use of crosses!
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"What kind of crosses are these?" Seva wondered aloud. "And why do they need to be saved?"

Just then, a schoolgirl with three funny pigtails fluttered out of one revolving door. It was little Three.

"Three, what’s in your suitcase?" Seva asked her.

"Hello!" replied the well-mannered Three.

"Oh yes, I completely forgot," Seva caught himself. "Of course, hello! Could you tell me what’s jingling in your suitcase?"

"Active signs." Three pointed to the sign: "It’s all written there. Can’t you read?"

"I can, but I don’t understand what these signs are and how they act?"

"Oh no, no. They can’t act by themselves. They only help others perform various actions."

"Stage actions?" Seva joked.

"Come on!" Three energetically shook her pigtails. "Not stage, but arithmetic!"

"I understand: addition, subtraction, multiplication, and division."

"And many others."

"What other ones?" Tanya was surprised. "Besides these four, there are no other actions."

"What are you saying!" exclaimed Three. "Besides arithmetic, there can be completely different actions — for example, algebraic ones."

"I don’t know of such," Tanya shrugged. "I’ve never even heard of them."

"Really?!" Three threw up her hands in amazement.

Crash! The suitcase fell to the ground, and all its contents spilled out. We hurriedly rushed to pick them up.

What wasn’t there! Dots and commas, small dashes, large dashes, crosses, round brackets, square brackets, curly brackets, and many more completely incomprehensible signs.

"Oh, how clumsy I am!" Three was upset. "Please be careful. These are very important signs. This small dash, for example. If you forget to put it between two numbers, then no one will guess that one number needs to be subtracted from another."

"That’s a minus!" Seva blurted out.

"Of course!" Three was delighted. "And if I place two such dashes one above the other, it will no longer be two minuses, but…"

"…an equals sign," Seva couldn’t resist saying.

"So you know everything! I think there’s no need to explain further. Here, for example, this cross…"

"That’s a plus," said Seva. "It’s needed for addition. But why do you have a notice saying 'Save on the use of crosses!'? Is it really so that people would add less?"

"Oh, what are you saying!" Three laughed. "Add as much as your heart desires! The thing is that the cross is used not only as an addition sign but also as a multiplication sign. You just need to stand it on both legs — like this: X. That’s why we don’t have enough crosses, and we decided to replace them with dots."

"But such a dot can easily be confused with a punctuation mark!"

"No, no!" Three waved her hands. "It’s very simple: our dot is placed a bit higher than a punctuation mark."

"And what is this?" asked Seva, pulling a funny figure from the suitcase. "A net for catching

butterflies?"

"How funny you are!" Three giggled. "This is also a sign. It’s used when extracting roots from numbers. And it’s called a radical."

"So numbers have roots, just like trees?" Seva was delighted.

"What horror!" exclaimed Three. "You understand everything literally."

"But what kind of roots are these, really?"

"Allow me to answer your question with a question: what is three times three?"

"Of course, nine!"

"Excellent! Without realizing it, you performed an important and beautiful action: you raised three to a power!"

"No," objected Seva, "I simply multiplied three by itself."

"Exactly. But that’s what raising to a power is. And in this case — to the second power."

"And can you raise to a third power too?" asked Tanya.

"Of course. For that, you need to multiply nine by three again."

"So three, multiplied by three and once more by three — that’s the third power of three?" said Tanya.

"Absolutely correct. Therefore, the third power of three equals…"

"…twenty-seven," Tanya finished.

"But you can do this endlessly!" said Seva.

"How correctly you noted that!" Three admired. "Precisely endlessly! And then fourth, fifth, sixth powers will be obtained..."

"Curious."

"But let's return to the beginning of our question," continued Three. "You asked what a radical is? Let's start from scratch. Three times three is nine. And now I'll ask you the same question from the end: what number needs to be raised to the second power to get nine?"

"Three," Seva answered immediately.

"You see, from nine we found out which number was raised to the second power. And this number turned out to be three."

"Is this the action that's called extracting a root?" asked Tanya.

"Yes!" Three was pleased. "And it's denoted by a radical."

"And you thought they catch butterflies with it," Tanya teased.

Seva solemnly raised his hand:

"I swear, now I will always remember what the root of nine equals."

"And yet," continued Three, "you shouldn't think that the root of nine always equals three! It all depends on which root you're extracting."

"What," Seva was taken aback, "are there different roots?"

"Completely different! There are roots of the third and fourth degrees. You'll learn about this in due

time. And now excuse me. I’m afraid I’ll be late for Good Wishes Square."

Three grabbed the suitcase and ran away.

And only then did we notice that Four with a bow had disappeared somewhere. After consulting, we decided to continue our journey alone. It wasn’t difficult: all the city’s residents were now moving in one direction.

Square of Good Wishes

It was an enormous field, completely filled with the inhabitants of Arabella. And, just like on the Avenue of Active Signs, perfect order reigned here.

At the entrance to the square, there was some amazing structure. My kids were examining it with delight, climbing up the steps, peering inside through round, colorful windows.

"Is it a rocket launcher?"

"No, it’s a spaceship!"

"I think it’s a nuclear power station!"

I remained silent: let them figure it out themselves.

Unexpectedly, a plump Eight, who was holding a little Zero by the hand, intervened in the conversation.

"Hello!" she addressed us.

"Hello!" repeated Zero after her and yawned deliciously.

Eight shook her head:

"What am I to do with him? He only fell asleep toward morning, and now he’s yawning. How can I let him go on such a serious journey?"

"Weren’t you the one singing to him: 'Sleep, little Zero, sleep tight'?" asked Tanya.

"Who else but me could sing a song that I composed myself? And weren’t you the ones walking under my windows at night?" Eight inquired in turn.

"Yes, yes, they were walking!" Zero rejoiced. "This girl," he pointed at Tanya, "asked how mailmen deliver letters to us if all houses have the same number."

"What does it matter who receives the letter," objected Eight. "Letters addressed to any of us equally concern everyone."

"And me, and me they concern!" shouted Zero.

"What a smart child!" Eight was touched.

"Since you’re so kind," Seva addressed her, "would you tell us, dear Eight, where did you get a son named Zero? I thought your children would also be eights."

"Of course, like all other eight-mothers, my children are also eights. And fives have fives, twos have twos, and so on. But zeros belong to everyone. Zeros are our adopted children. But we love them as our own, perhaps even more. They are so small, so defenseless. Without us, they mean absolutely nothing."

"Where did they come from?" Seva asked in surprise.

"Oh, that’s a very long story! You probably know that in our homeland, India, there were only nine digits. These nine elders formed the Arithmetic State. Now they sit on the Council of Elders and govern us. Soon, people decided that it was very inconvenient to manage without zeros. Well, think for yourself: you need to write the number 205, but you have only nine digits, no zero. What will you do? You’ll put a two in the hundreds place, a five in the ones place. But what will you put in the tens place? After all, there are no tens in this number! You can’t write the number 205 like this: 2NONE5! That would be terrible!"

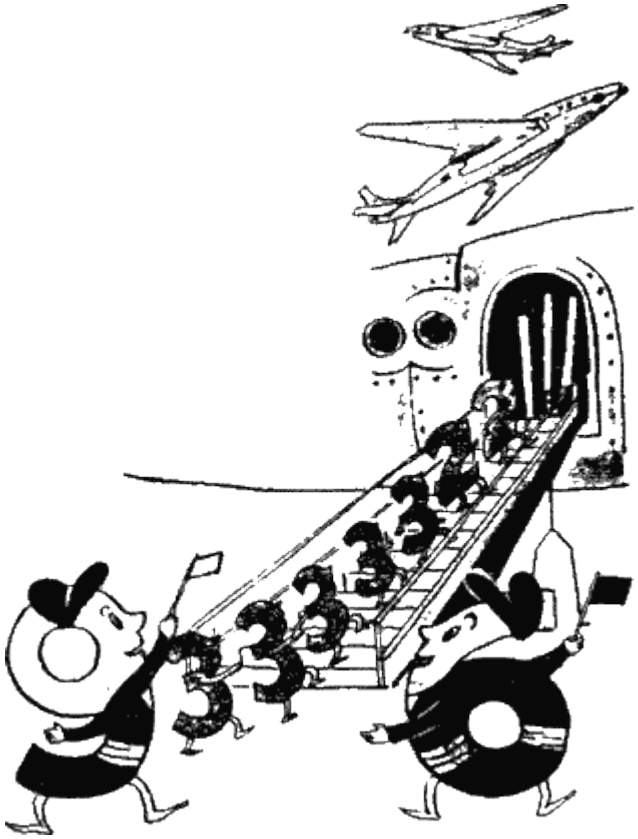
And people decided to put zero instead of the word "none." That’s how these sweet, lovely little ones appeared in our state, whom we gladly adopted. That’s where my round little son came from… Now go, go, my little one, or we’ll be late for the rocket. Say "goodbye" to everyone."

"Good-byyyeee!" sang Zero and rolled after his solid mother.

At this time, we heard a familiar voice:

"Here they are! I was already thinking I’d never find you." Before us stood Four with a bow. "Excuse me, please, I had to put my little ones on a high-speed airplane. Today they’re flying away for the first time."

"Strange," said Seva. "Where is everyone flying to?"



"What do you mean—where?" Four was surprised. "To you, to humans. To factories, to plants, to farms. To workers, accountants, scientists. And to schoolchildren, of course. We're expected everywhere: in villages, at polar stations, on ships sailing far, in space rockets. Every year we're needed more and more. We're being torn to pieces. Or rather, to all five parts of the world: invited to Asia, to Africa..."

Four didn't finish because at that moment dozens of powerful loudspeakers began to speak:

"Attention! Dwarves! In a minute, you will depart on a long journey to humans. Listen to the good wishes of the Council of Elders. Dwarf One is at the microphone.

"Dear friends, brave travelers, tireless workers! The Council of Elders wishes you a good journey and a safe return. We are confident that you will not disgrace our glorious state and will work honestly for the benefit of humanity. In the hands of good people, you will bring benefit; in the hands of evil ones, you may bring destruction. Serve good people, beware of the evil ones. Have a happy journey!"

Music started playing, and one by one, huge gleaming machines began to rise into the air. There were many of them, and each carried millions of dwarves. A delightful spectacle!

For a long time, we couldn't come to our senses and kept looking into the sky, where there was nothing left except light white clouds.

"Strange," Seva finally spoke, "so many flew away that you can't count them, but the crowd in the square isn't getting smaller. Maybe it just seems that way to me?"

"Oh, on the contrary, you're very observant!" noted Four. "The crowd indeed isn't getting smaller."

"How can that be?" Seva was puzzled. "If you keep taking one candy from a huge box, the candies

will eventually run out."

"Candies will certainly run out," smiled Four, "but dwarves—never."

"Nonsense!" Seva declared.

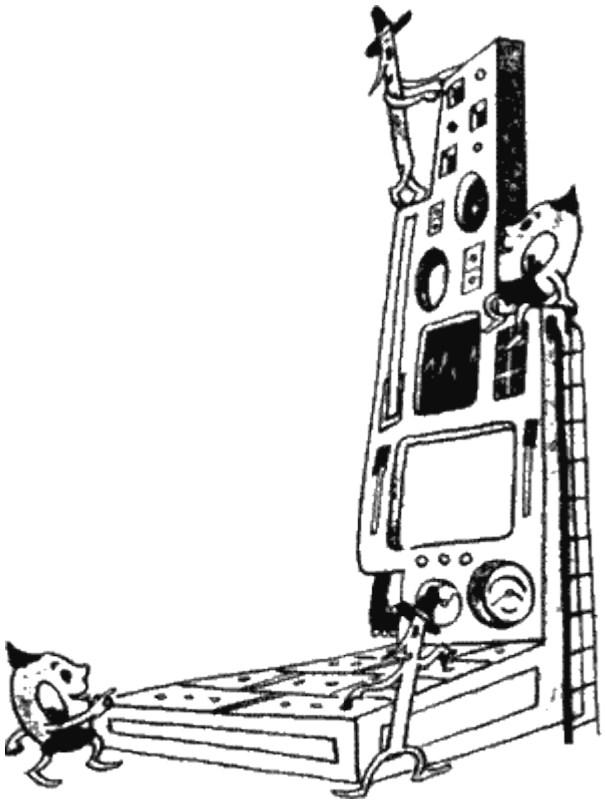
"That's rather weak evidence," Four remarked dryly. "In such cases, I usually end the argument. But this time I'll make an exception. I feel sorry for you because you don't know what infinity is."

"Why don't I know?" Seva took offense. "Infinity is when there's a lot of something. An awful lot!"

"No, no, no!" objected Four. "That's completely wrong. Even our zeros don't reason like that. Infinity is something that has no end. No end at all."

"Well, there must be an end somewhere?" Seva insisted.

"That's just it—nowhere! You think you've reached the very end, looked beyond, and there... And there's infinity again. And so on infinitely. You chase it, and it runs further and further away. You walk through a huge crowd of dwarves, you walk and walk until your legs can barely hold you, and ahead there are still as many people as there were before. And no matter how far you walk, you're always in the very middle of the crowd. Walk for a hundred, a thousand, a million years! That's what infinity is!... And please don't object!" she stopped Seva sternly, who had already opened his mouth. "If you don't understand, then I'll return to this question, because one can talk about infinity infinitely."



"Tell me, what is that huge building?" asked Oleg, trying to smooth over Seva's tactlessness. He pointed to the strange structure that had attracted our attention from the beginning.

"Not a building, but a machine," corrected Four, "an electronic computing machine. It's our great friend. There are many such machines in Dwarfland, and each is engaged in its own work."

"And what does this one do?" asked Seva. He recovered from his embarrassment surprisingly quickly.

"This machine performs the most precise calculations. It can count how many dwarves have gone on a journey, at what speed they need to fly to arrive on time. The machine itself will choose the shortest route; it controls the flight and helps to avoid all obstacles that the rocket encounters on its path. It’s a very smart machine!"

"It must be serviced by many dwarves."

"Only two digits, the two smallest digits: zero and one. But they handle their work excellently. Imagine, they have excellent memory. If they learn something, you can be sure: they will never forget it."

"Lucky them!" sighed Seva.

"And why do only zero and one work in this machine?" asked Tanya.

"No one else is needed. You know that zero is nothing. So it denotes the word 'no,' while one denotes the word 'yes.' It turns out that these two words are completely sufficient to solve any problem."

"How so?" Seva asked skeptically.

"Let’s play a game," suggested Four. "Guess what I have in my pocket? Ask me any questions, but ones that I can answer only with 'yes' or 'no.' Shall we start?"

The kids didn’t need to be asked twice. Questions poured one after another:

"Do you have something edible in your pocket?"

"No."

"A school item?"

"Yes."

"Liquid?"

"No."

"Solid?"

"No."

"Soft?"

"Yes."

"Long?"

"No."

"Round?"

"No."

"Rectangular?"

"Yes."

"Do you write with it?"

"No."

"Do you blot with it?"

"No."

"Do you erase with it?"

"Yes."

"An eraser!" said Oleg.

"Correct!" answered Figure Four. "See, you solved the problem just from my answers, using only two words: 'yes,' 'no.' That's how the machine works. Except it works very, very quickly. It's even called high-performance. We have a wonderful palace where these smart machines perform fast. It's the Palace of Cybernetics. Be sure to visit there. And now I invite you to the stadium—to have fun. A performance will start soon—ballet on ice. Little dwarves on skates! I recommend watching it."

Needless to say, with what joy we set off for the stadium!

Ice Ballet

It was a school morning performance, very similar to those we are used to back home. A good half of the audience consisted of excited mothers, aunts, and grandmothers of the little performers. They chatted animatedly and impatiently glanced at the large ice field, where decorations depicting a dense forest had just been set up.

The director—a slender, flexible Seven—gracefully crossed the ice stage and skated up to us, smiling warmly.

"Thank you for coming. It was I who asked my friend Four to bring you here."

We were seated in the front row, like honored guests.

And immediately, a blindingly bright light flashed over the ice field.

The conductor waved his baton, and to the sounds of cheerful music, two little ones ran onto the stage. They unfolded a long banner, and everyone read the title of the ballet:

```
MAGICAL TRANSFORMATIONS AND JOYFUL PERMUTATIONS!
```

The little ones ran away, and in their place appeared others, in colorful tunics, led by the kind Calculator-Builder.

He eloquently (though without words) described the great deeds they were about to accomplish for the benefit of people. The numbers, no less eloquently (also without words), expressed their complete readiness to follow the kind Calculator-Builder.

Suddenly, drums thundered in the orchestra, and the evil Calculator-Destroyer appeared on stage.

Upon seeing him, the numbers scattered in all directions. They were afraid he would take them captive and force them to work for him. And this villain was engaged in very bad deeds. He hated people and wanted to destroy them.

The kind Calculator-Builder stood to protect the frightened little ones.

Then the Calculator-Destroyer, seeing that he couldn't manage alone, called his army for help. And so warriors appeared in white uniforms with black crosses on their chests. They seized the kind Calculator-Builder, bound him tightly, and began to forcefully wedge themselves between the terrified numbers, who kept dodging them. This continued for quite a while.

To tell the truth, my kids didn't understand anything.

"Excuse me," Seva whispered to Four, "why are these numbers so afraid of the warriors with pluses on their chests?"

"Because right now they're still free digits. And when the pluses start adding them up, they'll turn into numbers. And then, whether they like it or not, they'll have to work for the evil Calculator-Destroyer."

"I thought," Seva objected, "that digits and numbers were the same thing!"

"Oh no! The difference between digits and numbers is the same as between letters and words. Words are made up of letters, and numbers are made up of digits. There are few digits and letters, but there are many words and numbers. You can perform various operations with numbers, but not with digits. When a digit becomes a number, this number can be given any name. Numbers can be called birds, books, apples, or they can be called rifles and cannons. That's what the evil Calculator-Destroyer is after. That's why the digits are so afraid of him."

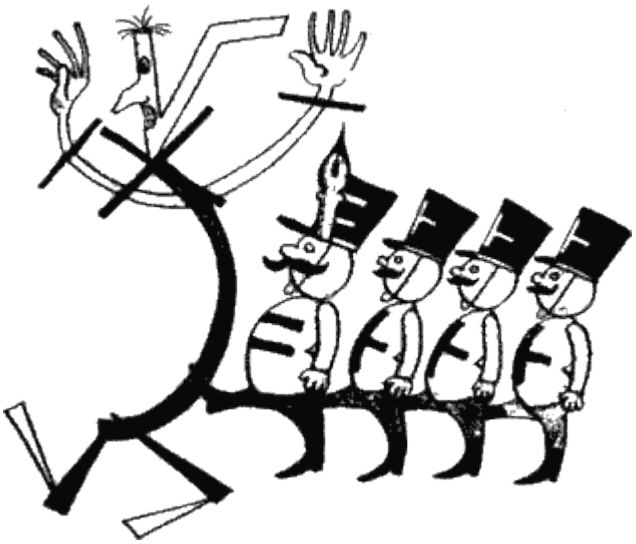
Meanwhile, with the help of his warriors, the Pluses, and their commander the Equals Sign, the evil Calculator-Destroyer finally forced the digits to arrange themselves as follows:

```
1 + 3 + 4 + 2 = 10
```

The poor digits, transformed into numbers, immediately froze. They were very sad. People in the audience were crying. And the Calculator-Destroyer was triumphant. He knew well that numbers would never dare to violate equality, would never dare to run away from him. After all, ten must always equal ten!

Now the numbers were in his power!

And suddenly (what fairy tale is without a magical "suddenly"!)... suddenly the little Zero from the number 10, an exact copy of our acquaintance, quickly stood on the other side of the One. And instead of ten, something odd appeared—01!



The audience gasped. This was unheard-of heroism. The Equals Sign immediately fainted—he couldn't bear such a violation. And the Calculator-Destroyer was so frightened that he ran to call the fire department, because in Dwarfland, firefighters are also called by dialing 01. Just like in our country.

The audience applauded loudly, and the digits quickly untied their kind leader and, in their joy, started a fun game, in which their recent enemies—the Pluses and the Equals Sign—took part. They too were tired of serving the evil wizard. From now on, they would only do good.

First, to the sounds of a smooth waltz, the numbers formed a group already familiar to us:

```
1 + 3 + 4 + 2 = 10
```

Then a magical spectacle began. Illuminated by yellow, then red, then blue spotlights, the young figure skaters began to change places, forming new and new groups:

```
3 + 1 + 2 + 4 = 10

2 + 3 + 4 + 1 = 10

4 + 1 + 3 + 2 = 10

1 + 4 + 2 + 3 = 10
```

and so on.

And only the ten, which stood after the Equals Sign, remained in place all the time. And the numbers in the dance bowed low to little Zero, and he stood pleased but modest, as befits a hero.

This continued until the skaters returned to their original position.

"How many times did they change places?" asked Seva. "I tried to count and got confused."

"Exactly twenty-four times," answered Four.

"Really that many?"

"If you doubt it, check for yourself," she smirked.

Here the first part came to an end. The performers lined up in a semicircle and, holding hands, sang:

> All’s well that ends well!
> But in this play, there’s a lesson to tell:
> From changing the order of addends
> The sum does not change at all!

First Encounter

During the intermission, we went backstage to thank Seven and all the performers for the interesting show.

This turned out to be harder than we thought. Backstage was terribly crowded, just like at our school amateur concert. Happy relatives were smothering the little performers with hugs and enthusiastically predicting great futures for them. Relatives always exaggerate a bit!

We still managed to reach Seven. We expressed our delight to her. She was happy and asked us to definitely watch the second part of the show.

"We’ll show you aerial **multiplication**. It’s the best number in our program!"

"Will Zero also participate in it?" asked Tanya, who really liked the little performer’s acting.

"Yes, of course. But this time he plays a supporting role."

"Why?" Tanya was disappointed.

"You can’t always play the lead roles," replied Seven. "Our performers must play all kinds of roles. I’ll tell you a secret: Zero was very offended by this. His mother convinced him that he’s a talent."

At that moment, a loud squeal was heard. The door to the costume room flew open. Someone whirled past us and disappeared into the crowd.

A terrible commotion began. The numbers scattered in all directions. Just like in the ballet when the evil Calculator-Destroyer appears.

Everyone was shouting chaotically:

"Help! Take it away from him! He will destroy us!..."

"I knew it! Naughty boy! He stole the multiplication sign." — And Seven bravely rushed into the

thickest part of the crowd.

"Who stole the multiplication sign?" we asked Four.

"Who else? Zero!" she squeaked, timidly hiding behind our backs. "The same one you were praising so much."

"And why does he need the multiplication sign?"

"He’s taking revenge because he wasn’t given the main role."

"Some revenge," laughed Seva, "stole a multiplication sign."

"Don’t say that!" Four exclaimed in horror. "Don’t you know what disaster little Zero can cause with a multiplication sign? If he stands next to any number — it immediately turns into nothing. In about an hour, all residents of Arabella will turn into zeros. Can you imagine a state consisting only of zeros?!"

A squad of firefighters ran past us. (As is well known, there are firefighters in every theater.) They had glasses with powerful magnifying lenses (for searching the smallest numbers). In their hands, they held long, thin rods. As it turned out, specifically for catching zeros.

Meanwhile, Zero ran out onto the ice field. Then the firefighters and a handful of desperate daredevils led by our glorious Seven surrounded the field in a tight ring and began to carefully approach the troublemaker.

A forest of long sticks was aimed at Zero.

But he was nonchalantly tracing patterns with his feet and sticking his tongue out at his pursuers.

It seemed like in another minute they would catch him. But not so fast!

The mischief-maker skillfully jumped up and climbed to the very top of a decorative tree.

"Drop the multiplication sign right now!" Seven demanded.

"No, I won’t!" giggled Zero, cheerfully swinging his legs.

"Then you’ll sit here until the end of time!"

"I’ll just jump over your heads. Right into the audience!" — And Zero pretended that he was about to carry out his threat.

A real panic started in the crowd. Spectators rushed in horror toward the exits. Bottlenecks formed at the doors. And then Seven rushed to the phone.

"Quick arithmetic help!..." she shouted into the receiver. "Is this Emergency?... A terrible disaster!... Yes, yes, it’s Zero again! Send giants immediately!"

"What kind of giants are those?" Seva asked Four.

"They are inhabitants of Infinity — infinitely large numbers!" she replied.

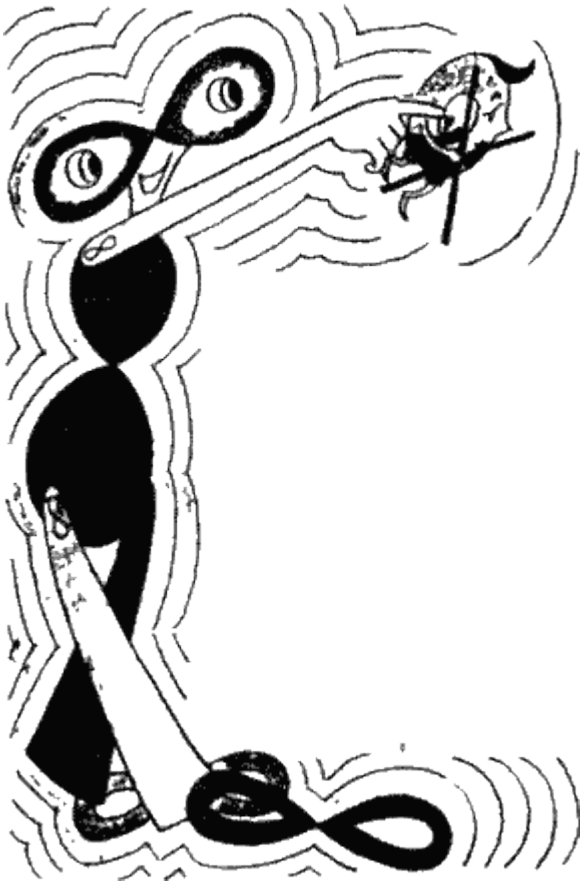
"And they’re not afraid of Zero with a multiplication sign?"

"Of course not! They’ve been given an anti-Zero vaccination! So when multiplied by zero, they don’t turn into zero themselves. Except maybe sometimes. For variety. When they feel like it. And then they can turn into any number of their choice."

The emergency help didn’t keep us waiting. Not even a minute had passed when a strong wind rose over the ice field, and from somewhere above came:

"Greetings from Infinity!"

A giant palm stretched out toward Zero, and he obediently placed the stolen multiplication sign on it. He knew that there was no joking with the Giant.



Two enormous fingers lifted Zero and carefully placed him on the ice.

"Hurray for the Giant!..." everyone shouted. "Long live the kind giants!..."

The Giant waved his hand amiably and disappeared as suddenly as he had appeared.

Zero sat on the ice, whimpering pitifully.

"Well," asked Seven, "how do you explain your behavior?"

"Forgive me. I didn't want to do anything bad," begged Zero. "I just thought I would scare everyone a little. Please don't kick me out of the drama club!"

"Well, we'll see about that!" Seven said sternly and... fainted. Now that all the excitement was over, her strength had left her.

We had to announce that for technical reasons the show could not continue.

That's how our first acquaintance with the Giant from distant Infinity took place.

Wonderful Signs

We barely convinced Four with a ribbon to go home. She needed to rest after so much excitement. After setting a time and place to meet us, she ran off, and we went to wander around the city.

Soon we came out onto a wide, bright street where beautiful houses made of plastic, glass, and aluminum stood. Colorful light advertisements made them look especially festive.

The street was called "Automatic." We approached a large building.

"Issuance and acceptance of sighs from three to four o’clock," Seva read.

"How are you reading?!" Tanya was indignant. "Not sighs, but signs!"

"Thank goodness! I was concerned," Seva rejoiced. "Signs are a completely different matter."

"Oh, different?" Tanya persisted, "Could you explain what this 'different' is?"

"Of course," Seva replied casually. "For example, sarcasm is a sign of a bad character!"

"And chattiness is a sign of stupidity!" Tanya shot back.

"Instead of arguing pointlessly, let’s go in and find out what signs they’re talking about," said Oleg.

There was no arguing with that: as always, he was right.

We found ourselves in a bright hall. At first, it seemed like there was no one there. Suddenly Seva tugged on Tanya’s hand and nodded toward a small Five standing by the wall.

In a quiet, uncertain voice, Five was saying something. But to whom? There was absolutely no one next to her!

And suddenly the voice of an invisible interlocutor sounded. It was like thunder from a clear sky. We involuntarily looked up at the glass ceiling.

The voice paused for a moment and immediately thundered again, obviously addressing us:

"Hello, people! We’re glad to see you! My name is Automaton. I’m teaching this worthy little dwarf how to divide whole numbers. She wants to become a teacher."

And then we saw that Five was standing by a huge machine that occupied an entire wall. In the middle glowed a silvery screen, surrounded by multicolored lights that flashed on and off. Inside, something clicked and crackled. Occasionally a thin melodious bell rang.

"Would you allow me to continue the lesson?" the Automaton politely inquired.

"Please do," Seva replied. "We’d also be happy to learn division."

"What, you don’t know how to divide yet? Hr-pr-tr! Excuse me, that’s my smallest gear turning in the wrong direction. You upset it."

"No, you misunderstood me, we actually do know how to divide."

"Ah, you do? Well, that’s quite different. Would you like to solve a little problem? I was just about to propose it to my young student."

Numbers lit up on the screen:

```
135,227 : 9 = ?
```

"Allow me," said Five. "Let’s start by the rules: first we divide thirteen by nine…"

"Hr-pr-tr! To answer my question, you don’t need to divide at all. I was going to ask you: is this number divisible by nine? Yes or no?"

"What, you want us to tell you right away, without dividing the numbers?" Seva marveled.

"Exactly!"

"But that’s completely impossible!" exclaimed Tanya.

"Why not?" the Automaton replied with dignity. "For this, you only need to look at which light lit up above the screen. Take a look."

"Red!" Five shouted.

"Well, there you have it. Since the red light lit up, it means this number is not divisible by nine. Now look at the screen again."

There was already a completely different number:

```
264,852 : 9 = ?
```

"And now the green light has turned on," Five reported.

"As it should be, because this number is divisible by nine."

"This is very simple," said Five, "red light – the number is not divisible by nine, green light – divisible by nine."

"Ha-ha-ha!" laughed the Automaton. "It’s simple because I’m the one turning on the lights. But try to turn on the right light yourself. Ha-ha-ha!"

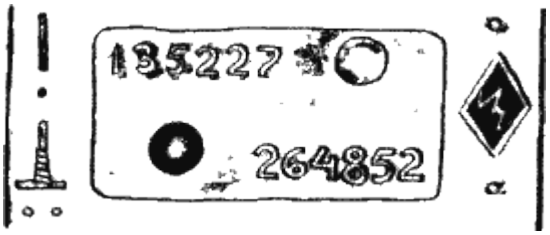
Little Five blushed to her ears.

"Well, don’t be upset, I was joking," the Automaton consoled her. "The fact is that numbers have signs by which you can determine at first glance whether they wish to be divisible by certain numbers or not. Unfortunately, I have a very small set of such signs. Therefore, if any of you find a new, unknown sign of divisibility, let me know immediately. That would be wonderful! You can’t even imagine what great benefit you’ll bring to people. I know the signs of divisibility by 2, by 3, by 4, by 5, by 6… Even by 10 and by 11. A few more signs – and that’s all!"

"Tell us about one of the signs," Five asked. "This is very interesting."

"In that case, let’s return to the two numbers I just showed on the screen. Let me remind you of them."

The numbers appeared on the screen: 135,227 and 264,852.



As you can see, each number consists of six digits. Let’s take these digits as numbers. And put plus signs between them.

On the screen, under the first number, a sum appeared:

```
1 + 3 + 5 + 2 + 2 + 7 = 20.
```

"Now tell me: is the number twenty divisible by nine? No, it’s not. So the whole number is also not divisible by nine. Let’s try the same with the second number."

The sum lit up on the screen again:

```
2 + 6 + 4 + 8 + 5 + 2 = 27.
```

"You see, we got twenty-seven. And this number is exactly divisible by nine. So the whole number is also divisible by nine. That’s the sign of divisibility by nine. It’s very easy to state it like this: a number is divisible by nine if the sum of its digits is divisible by nine."

"In that case," said Oleg, "I know the sign of divisibility by three. After all, nine is three times three! So if the sum of the digits of a number is divisible by three, then the number itself is also divisible by three."

"Absolutely correct! You will be a great mathematician!" the Automaton solemnly declared.

"I also know one sign: if the sum of the digits of a number is divisible by five, then the number is divisible by five," said Seva. He also wanted to become a great mathematician.

"By no means, by no means!" exclaimed the Automaton, indignantly flashing all its lights. "Tr-pr-hr! How can you measure everyone by the same standard? The number twenty-three is not divisible by five, although the sum of its digits equals five. The sign of divisibility by five is very simple: only those numbers that end with five or zero are divisible by five. For example, 75, 210, 625, 4,168,596,895, and so on."

"How simple!" Tanya laughed.

"There are more complex signs. For example, the sign of divisibility by eleven."

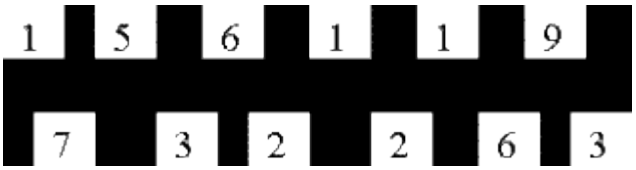
"Oh, please, tell us about this sign!" Five requested.

"Good. Listen to me carefully. Let’s take the number

```
175,362,121,693.
```

"Wow!" the children exclaimed. "This number is even hard to read."

"Hr-pr-tr! One hundred seventy-five billion three hundred sixty-two million one hundred twenty-one thousand six hundred ninety-three!" the Automaton rattled off in one breath. "Nothing special. Let's see if it's divisible by eleven. Let's arrange the digits of this number like this:



We'll place every second digit a bit lower. And now let's put plus signs between the digits in each row. We get:

```
1 + 5 + 6 + 1 + 1 + 9 = 23,

7 + 3 + 2 + 2 + 6 + 3 = 23.
```

In both rows, the sum of the digits is the same. And this means that the number is definitely divisible by eleven."

"Really?" Seva doubted.

"Check it," the Automaton suggested.

"That would take too long," Seva replied.

Then Oleg showed us a page from his notebook where he had already performed the division.

"Absolutely correct!" said the Automaton. "You really will be a good mathematician."

And on the screen, numbers flashed:

```
175,362,121,693 : 11 = 15,942,011,063.
```

"Here's your answer: fifteen billion nine hundred forty-two million eleven thousand sixty-three."

"So, only such numbers where the sum of digits in odd positions equals the sum of digits in even positions are divisible by eleven?" asked Oleg.

"No, not only these numbers are divisible by eleven. There's a more general sign of divisibility. For example…"

At that moment, a prolonged bell rang, signaling the end of the work day. The Automaton barely had time to say goodbye before all its lights went out. What a pity!

We went out into the street. Now we had to hurry to the Square of Joys and Sorrows, where Four with a ribbon had arranged to meet us.

On this square, air ships returning from the world of humans to Dwarfland landed daily.

An Unexpected Sorrow

We arrived on time. The first rocket had just landed. They lowered the ramp, and thousands of travelers found themselves in the arms of their relatives and friends.

Next to us, a happy mother-Five was hugging her youngest daughter.

"I missed you so much!" she complained.

"And I had so much fun!" chirped the daughter. "We were sent to an architect—he was commissioned to build houses in a new city. He thought for a long time: how many floors should these houses have? First he considered Four, then Nine. But then he looked at me and said, 'The houses will be five stories high!'"

"Oh, you're my beauty!" the mother gushed, but then gasped: "Where did you get so dirty? Some beauty you are!"

"A clumsy draftsman spilled a bottle of ink on me. I've been rubbing and rubbing with an eraser, but it won't come off."

Three dwarf figures walked by importantly, singing loudly:

"Tu-tu-tu… tu-tu-tu!" These were the numbers 1, 0, and 4.

"Why are they being so self-important?" asked Seva.

"Don't you recognize these three heroes?" responded a female dwarf walking behind them. "Their portraits were displayed today on a jet airplane. My son is among them. This airplane is called 'Tu-104.' Now all they do is sing: 'Tu-tu-tu… tu-tu-tu!'"

"And I was at the circus," a tiny Two was telling her mother. "I saw a man in a red wig doing a double somersault. Can I try doing a double somersault at home too? After all, I'm Two—I should be able to do it."

"I'll spank you," replied her mother, "and you'll stop thinking about your somersault! Do you want to break your head?"

The engines roared again, and a large passenger plane descended onto the square. Passengers began to exit. They were pressing small white handkerchiefs to their eyes.

Immediately, everyone in the square stopped laughing and became sad. The Square of Joys transformed into the Square of Sorrows.

"What a misfortune! What a tragedy!" lamented a Nine, coming down the ramp. "Poor little Zero is gone, gone! This morning at the Square of Good Wishes, we had forty-three little Zeros with us. Then we were sent to school, to the third grade. It was so nice there, so much fun! The

schoolchildren were learning to divide whole numbers. We ran from desk to desk, from notebook to notebook. But then, when we got on the plane to return home, we found we had only forty-two little Zeros! One was missing. What a tragedy!"

"It’s my son who’s missing!" wailed a familiar fat Eight. She had already looked through all the arriving Zeros. "Why didn’t I go with him? What will I do without him?"

"Maybe he’ll still return? Maybe he got on the wrong plane by mistake?" they consoled the poor mother.

"Or perhaps," said our Four with a bow, "he never left here at all? He’s such a mischievous one! He might have hidden during the morning boarding and then run off to the movies."

"No, more likely," suggested some One, "to a football match."

"Or maybe he’s at the circus doing a double somersault?" said the tiny Two.

"My poor son! Where are you now?" the mother-Eight wouldn’t calm down.

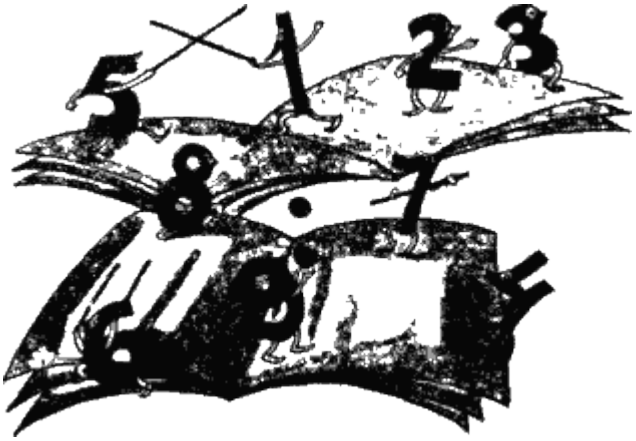
"Calm down," said Four, "your Zero will be found. Morning is wiser than evening. If he doesn’t appear today, tomorrow we’ll turn the entire Arithmetic country upside down and find him for sure."

At that moment, they announced the arrival of a plane with the little ones. Four anxiously counted her charges and sighed with relief. They were all safe and sound.

Together we all returned. Along the way, the little ones talked over each other about their adventures.

We reached the Number Square, where we wished each other good night and parted ways.

Thus ended our first day in Dwarfland.



Second Day

Simplicity…

When we woke up in the morning, we discovered that Seva had disappeared. Since everyone knew his restless character, no one was particularly worried.

We were right. After some time, he came running back, upset: little Zero was still nowhere to be found!

Seva had deliberately gotten up early to inquire about the missing little one in the city.

"Let’s set out to search right after breakfast," Tanya suggested.

"Right!" Seva rejoiced. "I heard there’s some place in Dwarfland. It’s called Rome."

"Why do you call it 'some place'? Rome is a city in Italy," said Tanya.

"There’s one Rome in Italy, and another in Dwarfland!" Seva cut her off.

"Rome is an ancient state," said Oleg. "It no longer exists, but perhaps remnants of Rome have been preserved here."

I listened without intervening in the conversation. Seva asked me:

"Did little Zero end up in Rome?"

"He couldn’t have gone there," I replied, "he has absolutely nothing to do there."

"How do you know?!" Seva fumed. "If we’re searching — we should look everywhere."

"Well, I don’t mind," I agreed. "Besides, we’ll get to know the inhabitants of this 'place'."

We crossed Number Square, walked a bit along Automatic Street and turned left.

Before us was an endless alley. At its entrance sat a very old dwarf looking through a telescope.

"Not visible, still not visible…" he muttered to himself.

"What’s not visible?" Seva was interested. "Let me take a look. Maybe I can see it."

"How can you possibly see something that’s not visible? The end is not visible! Just yesterday I noticed a huge number at the very end of the alley and thought: 'Well, that’s it now. Nothing can be beyond that.' But today I looked: behind that number there’s another number, even bigger than

yesterday's!"

"What kind of number is it?" asked Tanya.

"As if I'd explain it to you just like that! How impatient you are! Better walk down this alley and look with all your eyes. Perhaps then you'll understand. Perhaps!..." — And the old grumbler buried himself in his telescope.

We walked along the left side of the alley and suddenly heard a command:

"Count off in order of your numbers!"

"Is this a morning roll call?" asked Seva.

The numbers standing on the left side began to shout:

"Two, three, five, seven, eleven, thirteen..."

The voices became increasingly muffled, fading into the distance.

"This is no longer order, but disorder of numbers," Tanya remarked.

However, the numbers named themselves exactly in the sequence in which they stood:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37 and so on.

"What crazy numbers are these?" Seva wondered.

"You're the crazy ones!" the old dwarf was indignant. "And ignorant too. Haven't you read the inscription at the entrance?"

"No," Seva faltered.

"This is the Alley of Prime Numbers! Understand?"

"And what are prime numbers?"

"Look to the right," said the dwarf, "perhaps this will clear your minds."

On the right side of the alley stood completely different numbers: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27 and so on.

"These are exactly the numbers," said Tanya, "that are missing from the left side of the alley."

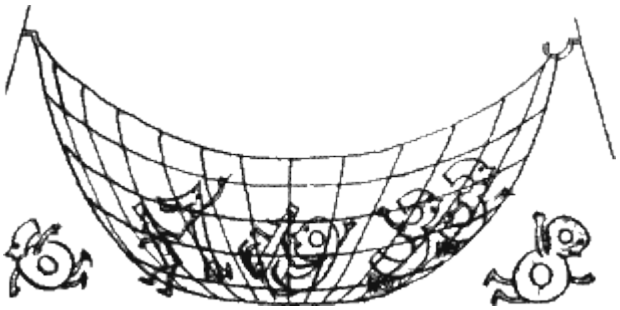
"And they’re not allowed there!" the dwarf giggled. "These are composite numbers, not prime ones."

"Why are they kept here?"

"I think my liver is starting to hurt from your absurd questions! Don’t you see what’s above you? You shouldn’t just look at your feet; sometimes it doesn’t hurt to look up."

We raised our heads.

"A volleyball net!" Seva gasped.



Indeed, a gigantic net was stretched over the entire alley.

"Again you’ve said nonsense!" the dwarf grew angry. "What does volleyball have to do with this? These aren’t games! And that’s not a net at all, young man, but a sieve!"

"A sieve?! What do they sift through it?"

"Numbers! They sift numbers!!" the dwarf shouted, losing all patience. "Look how thoroughly they’re being shaken! All kinds of waste, like composite numbers, fall through the sieve, and they’re directed to the right side of the alley. And in the sieve remain, in their purest form, our precious, our beloved prime numbers. They are carefully arranged in order on the left side of the alley. Look, aren’t they charming?" he suddenly became emotional.

The children nodded politely, although none of them found any charm in prime numbers.

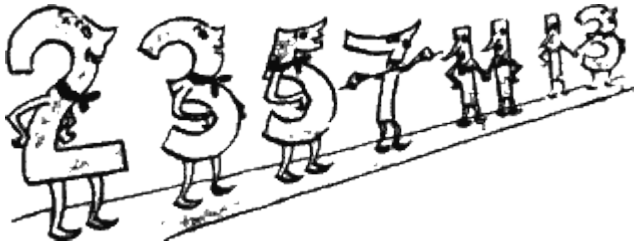
Fortunately, at that moment, faithful Four with a bow caught up with us. Everyone was loudly delighted.

"What a mean old man!" Seva complained. "All he does is grumble…"

"Not at all!" Four laughed. "He’s the kindest dwarf in the entire state! He just doesn’t like to show it. But let’s not distract the old man from his work. I’ll tell you everything myself."

We gladly sat down on a bench. And Four with a bow began her story:

"Long ago, people noticed that there are numbers that don’t recognize anyone but themselves. They aren’t divisible by any number other than themselves. And they make an exception only for one. And only because division by one doesn’t affect them at all: after division by one, they remain exactly as they were before. These numbers people called prime, though it’s not so simple to find them among others. More than two thousand years ago in Greece, the famous mathematician Eratosthenes invented a very ingenious way to seek out prime numbers. He proposed using a special sieve through which all unnecessary numbers would be sifted, and all the needed ones — the primes — would remain."



"Just like panning for gold," said Oleg. "The sand washes away, and the gold remains."

"A wonderful comparison!" exclaimed Four. "Prime numbers are indeed our gold. So," she continued, "the wonderful sieve was named the Sieve of Eratosthenes. Now let’s see how it works. Let’s write down all numbers starting from two, up to… Well, I said 'up to' without thinking. After all, there’s no end to numbers. So, let’s arrange the numbers in order, starting with two:

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23 and so on.

Such a sequence of numbers is called the natural number series. Let’s discard from this series those numbers that are definitely not prime, that is, those that are divisible not only by themselves but also by other numbers. First, let’s sift out numbers that are divisible by two. What numbers are these?"

"I know," said Tanya. "All even numbers are divisible by two."

"Correct. Let’s sift out all even numbers except two, and then we’ll have this:

2, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41 and so on.

Now let’s sift out all numbers that are divisible by three.

These are 6, 9, 12, 15, 18, 21… But we’ve already discarded all the even ones — 6, 12, 18… What will remain in the series now? Here’s what:

2, 3, 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37, 41, 43, 47, 49, 53…

You see, fewer and fewer composite numbers remain in the sieve.

And then we’ll discard all numbers that are divisible by five, then those divisible by seven… Gradually, composite numbers will drop out of the natural number series, and prime numbers will remain, that is, those that are divisible only by themselves and by one.

Now we already know many prime numbers.

Here are the first of them:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97…

These numbers, as you can see, stand on the left side of the alley.

"Very simple!" declared Seva. "At home, I'll also create such an alley and write out all prime numbers…"

"Don't rush," Four interrupted him. "It's not so easy to write out all prime numbers. After all, the larger the number, the more difficult it is to determine whether it's prime or composite. If we knew the order in which they follow each other, that would be wonderful! Unfortunately, no one has yet been able to establish this order. Sometimes prime numbers stand right next to each other, they're then called twins, sometimes between two nearest prime numbers there's a huge distance, and it's completely filled with composite numbers. People have gone very far down this alley, they know many prime numbers, and still not all of them!"

"But maybe there are no more prime numbers further on?" Seva doubted.

"No! That can't be!" replied Four. "Long, long ago, one great scientist, also a Greek, Euclid, a predecessor of Eratosthenes, proved that there's no end to prime numbers. That's why our kind dwarf is so concerned! He has a lot of work. Just yesterday at the end of the alley, he saw a huge prime number, and today behind this number stands an even bigger one: 170 141 183 460 469 231 731 687 303 715 884 105 727. And tomorrow, maybe a new one will appear if people calculate it. And so on without end. Enough to lose one's head over. And we could talk about this endlessly too… Let's better focus on searching for poor little Zero," Four concluded her story.

"We're actually going to Rome for that," said Seva.

"To Rome for little Zero?!" Four was surprised. "He can't possibly be there!"

"We're still going!" Seva persisted.

"As you wish!" our guide agreed. "A guest's desire is law for us."

…And Perfection

We turned onto a small street.

"What a charming street!" Tanya clapped her hands.

"But this is Perfection Street," explained Four. "Very few numbers live here. But all of them are perfect. That's what they're called — perfect numbers. Unlike prime numbers, they definitely divide by various other numbers."

"So they're composite?" asked Tanya.

"Absolutely composite. But special. Perfect numbers are equal to the sum of the numbers they are divisible by. Except themselves, of course. Let's take a perfect number — 6. What numbers does it divide by? By 1, by 2, and by 3. Now let's add these three numbers:

```
1 + 2 + 3 = 6
```

"Amazing!" exclaimed Tanya.

"Or here's another perfect number — 28," continued Four. "Remember its smaller divisors?"

"We remember," answered Tanya. "1, 2, 4, 7, and 14."

"Add them up:

```
1 + 2 + 4 + 7 + 14 = 28.
```

"Cool!" shouted Seva.

"Aha!" Oleg realized. "So perfect numbers are equal to the sum of all their smaller divisors."

"Well done!" Four praised him.

"Are there many perfect numbers on this street?" Seva inquired.

"Unfortunately," Four sighed regretfully, "only twenty-four: 6; 28; 496; 8,128; 130,816... Beyond that, they grow faster and faster, and calculating them becomes more and more difficult. This street is still being populated. If you happen to find a new perfect number, tell it that it's eagerly awaited here."

"I never thought there were so many interesting numbers in Dwarfland," Seva said thoughtfully.

"Oh, this is just a tiny fraction of our riches!" Four replied proudly. "Many don't have enough lifetime to meet them all. For example, not far from here live inseparable friends. They love each other so much that they share everything they have. These are the numbers 220 and 284. They are remarkable because each is equal to the sum of the smaller divisors of the other. What are the divisors of 284? 1, 2, 4, 71, 142. And the divisors of 220 are: 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110. Let's try to add up the divisors of each number:

```
1 + 2 + 4 + 71 + 142 = 220,

1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284.
```

That's why these numbers are called amicable.

No wonder the famous Greek mathematician Pythagoras said: "A friend is a second self!" — and referred to the numbers 220 and 284.

And there are many such number-friends!

Here a conversation about friendship and loyalty began. And we didn't notice how we ended up outside the city.

The Ruins of Rome

We walked for quite a long time until Rome finally appeared on a hill. It was surrounded by ancient, half-ruined fortress walls. Below them was a moat, once filled with water, but now dried up and densely overgrown with weeds. A rickety wooden bridge was raised. The crooked gates were locked. They were guarded by a decrepit she-wolf.

"The local Romans claim," Four said, not without humor, "that this is the great-great-great-granddaughter of that very she-wolf who nursed the two twins: Romulus and Remus—the founders of ancient Rome."

"Look, look," shouted Seva, "they have a goose on their tower!"

"Perhaps it’s also the great-great-great-grandchild of those geese that saved Rome?" suggested Tanya, examining the amusing weathervane with interest.

"How could geese possibly save Rome?" Seva protested.

"Very simply," answered Oleg. "The geese started honking just when enemies were approaching the sleeping city. The warriors woke up and drove away the enemy."

We cautiously approached the moat. To tell the truth, we were intimidated by the she-wolf.

"Don’t be afraid," Four smiled, "she hasn’t bothered anyone for a long time."

And indeed, the she-wolf opened her mouth wide and… yawned.

They had evidently noticed us in the city. Some skinny figure, resembling a matchstick, peeked out from a wide crack in the gates and immediately disappeared. Following it, other matchsticks began to peek out.

After some time, a matchstick with some kind of long tube appeared on the tower. It put the tube to its mouth, and… two frightened mice flew out of the tube. Hoarse sounds were heard, reminiscent of a donkey’s bray.

After this, with incredible creaking and grinding, the bridge across the moat slowly lowered—exactly like an ancient old man whose joints had fused from long immobility.

Meanwhile, strange bustling was happening behind the gates. It seemed as if someone was trying to open a rusty lock with a huge key, but without success.

But finally, the fragile hinges couldn’t hold—the gates, without opening, fell flat to the ground, and we saw a large square.

Grass was pushing through the stone slabs. We were hit by the smell of mold and desolation.

"Nothing to be done—antiquity!" sighed Seva. But what was this?

From around the corner appeared four half-dead horses, pulling a very strange contraption on two enormous wobbling wheels. On both sides of this wreck stood whole armies of matchsticks, guarding their leader—an old man hanging on crutches. His feet, twisted with gout, touched each

other at the toes. His entire absurd figure resembled the letter "M."

The old man addressed us with a long, pompous speech in Latin, from which we understood only that we were being invited to enter the city.

"Go on," said Four, "and I'll wait for you here."

"What, you're leaving us?" the children were disappointed.

"It's better for me not to go there," Four explained. "The Romans don't like the inhabitants of Arabella. They're envious of us. People rarely use Roman numerals, while we're always in demand."

We entered the city. It was neglected and poor.

"I thought we would see the Colosseum," said Tanya disappointedly, "gladiators, lions, but here..."

It immediately became clear that the Romans couldn't speak our language. They bustled about and began looking for an interpreter. They had only one, and they couldn't wake him up.

Finally, they brought a sleepy matchstick, who kept yawning for a long time. This was the interpreter.

After many ceremonies accompanying the introduction, Seva finally asked the most important question:

"Do you have little Zero here?"

"Please repeat that once more," the interpreter requested. "I didn't hear you!"

"I’m asking: do you have little Zero here?"

The interpreter smirked contemptuously:

"Which Zero? You’re probably talking about that little circle who lives in Arabella for no apparent reason and represents absolutely nothing? No, no, we don’t have zeros! They’re completely useless. Besides, you can never tell where they begin and where they end. We Romans recognize only straight lines. It’s very convenient. You can immediately see where the feet are, where the head is."

"How do you form numbers then, for example, ten, one hundred, if you don’t have zeros?"

"All of this can be represented with just sticks."

"Even large numbers?"

"Even large ones. Watch."

The interpreter clapped his hands, and the matchstick warriors standing in the square instantly formed several orderly rows.

"Like athletes in a stadium," noted Seva.

"Each of these warriors," the interpreter explained, "is a unit. Nothing more. But from these units, I can compose anything I want. Now I’ll make them turn into twos. One, two!" he commanded.

A regrouping occurred in the square. All the matchsticks arranged themselves in pairs.

"Now you see before you the number two. Let’s continue. One, two, three!"

Before we could blink, there were three matchsticks in each row.

"There’s the number three for you," said the interpreter.

"And four?" asked Tanya.

"First, get acquainted with our five," the interpreter answered mysteriously and gave another command.

The matchsticks regrouped again in twos, moved close to each other, and leaned apart.

We saw a figure that we usually call a checkmark—V.

"Now it’s not difficult to get four or six," continued the interpreter. "If we put a stick to the left of five, we get four—IV; if we put it to the right, we get six—VI."

"So the whole point," Tanya realized, "is either to subtract one from five or to add it. If the unit is on

the left, it means subtract it; if on the right, add it."

"I understand!" exclaimed Oleg. "If you add two sticks to the right of five, it will be seven, and three sticks—eight."

"That's exactly what we do. See how simple it is," the interpreter said proudly.

"Then I know how to get nine," declared Seva. The interpreter looked at him mockingly:

"You're not planning to add four sticks to five, are you? Many make that mistake. Meanwhile, we represent nine differently. After all, it's closer to ten than to five. So it's simpler to put a unit to the left of ten—there's your nine!"

"But how do you represent ten?" asked the discouraged Seva.

The interpreter gave a sign, and the matchsticks transformed into nimble acrobats. Some fives flipped upside down, and others easily jumped on top of them—X.

"Awesome!" exclaimed Seva.

"Beautiful and simple!" confirmed the interpreter. "And then our usual rule: a unit on the left—nine, IX; a unit on the right—eleven, XI. Then XII, XIII, XIV, XV, XVI… Then two tens—twenty, XX; three tens—thirty, XXX…"

"Four tens—forty," Seva continued in the same tone.

"Stop!" said the interpreter. "I forgot to tell you that, besides sticks, we have four Latin letters: M, D, C, and L. M is one thousand and, as the largest number, our leader. His assistants are: D—five

hundred, C—one hundred, and L—fifty. So: forty is fifty minus ten. Therefore, it’s represented like this: XL. Let’s say you want to get the number 1663..." The interpreter bowed low, summoning the necessary letters.

We had to wait quite a long time: the elderly pensioners moved slowly. With difficulty, they formed the intended number: MDCLXIII.

"As you can see, we get along perfectly well without Zero!" the interpreter noted sarcastically.

"I think this is very slow and inconvenient," said Tanya. "Now I understand why people don’t use you anymore."

"You’re mistaken," replied the interpreter, turning red with indignation. "Just yesterday we were invited to the ninetieth anniversary of your respected scientist. We graced the head table all evening—XC—and listened to long laudatory speeches about the jubilee celebrant. Meanwhile, the celebrant himself often looked at his family heirloom watch, where only Roman numerals were on the dial. Then the scientist was presented with a luxurious edition of his works. And what do you think? All chapters were marked only with Roman numerals, do you hear—Roman numerals!"

"Well, for an anniversary you might still be useful," Tanya remarked, "but performing complex calculations with you is very inconvenient. After all, you can’t even be added or multiplied in columns, not to mention division. How do you do this?"

The interpreter pretended he hadn’t heard Tanya’s words. He was again seized by nervous yawning.

There was nothing more to see in this city, and we left it, saying goodbye rather coldly.

Evidently, everyone was offended at us, because even the she-wolf turned away when we descended the little bridge, and the goose had a sullen and angry look.

Four with a bow was waiting for us at the gates.

"You’re so beautiful!" Tanya hugged her. "And how nasty those burnt matchsticks are!"

"So you didn’t like it there?" Four brightened. "I must admit, I’m very glad about that. Still, it’s not worth quarreling with them. You’ll still encounter them on your way."

"Arab numerals are enough for me!" said Seva. "I don’t understand why people invented some other ones?"

"Many peoples had their own numerals," Four answered, "most of which you won’t find anywhere

now."

"What were these numerals like? What did they look like?" Tanya was interested.

"Would you like to see?... Archaeological excavations are being conducted nearby. Maybe we'll find something interesting?"

"And maybe that's exactly where little Zero is hiding?" the children suggested.

"Alas!" Four sighed. "He can't be there either. Still, let's go for a little while. It's very curious."

We enthusiastically agreed—after all, this was our first archaeological expedition.

Interesting Findings

Walking was not easy. Huge pits appeared here and there, with piles of rubble and earth towering beside them. Everywhere we saw dwarves, as industrious as ants. They dug in the soil with such pleasure, as if it wasn't hard work, but a fun game. And why not! Isn't it interesting to reconstruct the past of your state from rusty and mossy remains of antiquity that have lain in the earth for millennia!

We stopped at one of these pits and curiously observed the work. Just at that moment, a dignified dwarf pulled some small objects from a pile of earth.

"Oh, what cute little brooches!" shouted Tanya. She was a girl after all.

The dwarf smiled:

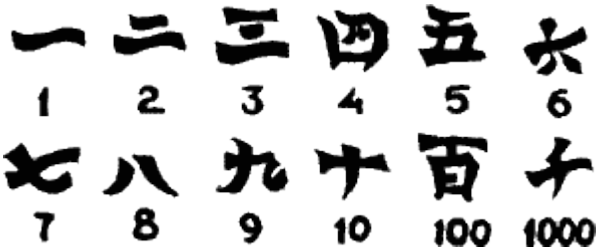
"What did you call them? Brooches? Such brooches were used in Ancient Egypt to represent words. They are called hieroglyphs. Once hieroglyphs were very complex, then they were simplified, but there were more and more of them. Hieroglyphs were also used to designate numbers."

"Yes!" Seva thoughtfully scratched the back of his head. "If only they used hieroglyphs for grades in the diary! Mom would never guess that I answered poorly!"

"For that, you would need to go to Egypt," smiled the dwarf.

"Or to China," added another dwarf who was standing nearby. "Hieroglyphs have been preserved there too."

And he showed us Chinese hieroglyphs representing the first ten numbers:



"But the most amusing hieroglyphs were still in Ancient Egypt," said the first dwarf, handing us a fragment.

"A bird!" Tanya exclaimed in delight.

100.000

"This bird represented the number one hundred thousand for the Egyptians. And this little man," he showed another fragment, "means a million."

1000.000

"How dreadful!" sighed Seva. "I don't envy Egyptian schoolchildren! It's hard enough to deal with Arabic numerals, but they must have had it really tough."

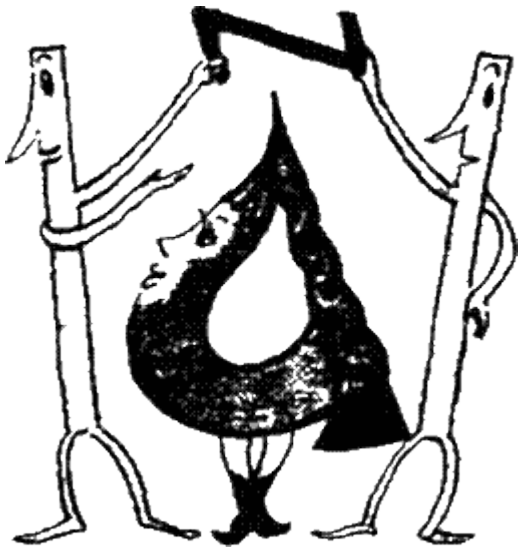
We thanked the dwarves and headed to the next group of archaeologists.

We were already quite close to them when Seva stumbled over some piece of metal. He pulled it out of the ground and began to examine it like a real researcher.

"Look at this interesting squiggle!"

"That's not a squiggle at all," Four politely said, looking at his find. "It's a titlo."

"You mean a title," Seva corrected. "Before the revolution, titles were given to all sorts of rich people — count, baron…"



"Almost right!" Four laughed. "Ancient Slavs marked letters with a titlo. When a titlo was placed above a letter, the letter turned into a number."

"So Seva is right," said Oleg. "Letters were given titles. Only the titled letters became not counts or barons, but numbers."

"Look at this tablet," said a dwarf who had heard our conversation. "It shows Slavic letters with titlos. Under each one is written the name of the letter and the number it represents."

"But how do you write numbers that aren't here?" asked Tanya. "For example, twelve?"

"I know," said Seva, "ten and next to it two. Like this:

"It's actually the opposite," objected Four, "first two, and then ten. And it was read as: two on ten. Interestingly, this order of reading numbers has been preserved to this day: twelve, fifteen — despite the fact that we now write the tens first, and then the units."

"This way of writing small numbers might be easy," said Seva, "but how do you write a large number?"

"Like this," the dwarf interjected into the conversation and showed several identical green copper badges:

This symbol denoted a thousand. The symbol was placed in front of the number of thousands.

For example,

represents twenty,

and like this — it’s already twenty thousand. Two such symbols denote a thousand thousand, that is, a million. This is already twenty million.

"But I must note," said Four, "that the ancient Slavs did not know numbers greater than a thousand. And when they became acquainted with the number ten thousand, it seemed so enormous to them that they began to call it 'darkness' (t’ma)."

"That’s probably where the expression 'pitch darkness' (t’ma-t’mushchaya) comes from," said Oleg. "It’s when there’s a lot of something!"

"So much that it’s dark in your eyes," added Tanya.

"However, later," continued Four, "the Slavs learned to count beyond ten thousand. First, they reached a million and began to call it 'darkness':

"And then they reached a million millions. This was their 'legion'."

"And further?"

"And further came the legion of legions — 'leodr'."

"And did they know the leodr of leodrs?"

"They did, and called it a 'raven'."

"Just like the bird," Seva laughed.

"That makes sense," Oleg inserted, "a raven is black, darker than darkness."

"And what was their name for the raven of ravens?"

"They didn’t have one," answered Four. "Greater than a raven, they said, is not for the mind to comprehend."

"So that’s the limit!" said Seva.

"Not quite," our guide answered. "In one manuscript, a number larger than a raven was found — ten ravens. And this number was called a **koloda** (block)."

And in that manuscript it says: "There is no number greater than this."

"So they stumbled upon this block and went no further," concluded Seva.

"But we will go further," smiled Four.

Along the way, another pleasant surprise awaited us.

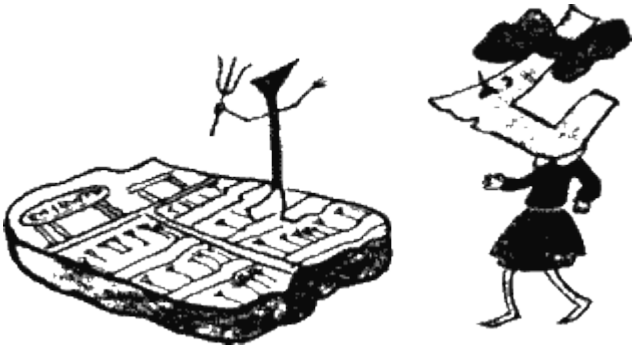
Oleg’s shoelace had come untied. He bent down to tie it and noticed that he was standing on a clay tablet. He cleaned off a layer of soil from it. And everyone saw that the tablet was covered with many rather deep dash-wedges.

"This is probably some ancient writing," decided Oleg.

"You’re not mistaken," replied Four. "This is cuneiform. This is how they wrote in Ancient Babylon. With small pointed sticks, the Babylonians pressed their writings into wet clay, and then baked the clay tablets in the bright sun. It was difficult to write intricate figures with sticks. Therefore, Babylonian writings consisted of small wedges."

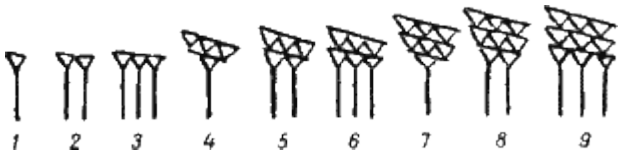
"Tell me," asked Seva, "did they also write numbers with letters in Babylon?"

"No," answered Four, "the Babylonians, unlike the Slavs, had numerals with which they wrote numbers. The numerals were depicted in the form of a thin stick with a small triangle on top:



"Just like a nail! With a head!"

"Indeed, it looks like a nail," agreed Four. "Only a nail has one head, but numerals could have many. This is how the nine Babylonian numerals were written:



"Look, the nine has a whole hat shop!" Tanya rejoiced.

"They’re very easy to count, these heads," said Oleg.

"That’s because there are no more than nine of them. But forty triangles would probably be hard to count," replied Seva.

"But why would you need to count forty triangles?" Four wondered. "After all, they had a different, simple sign for the numeral ten. Like this:

If they needed to write twenty, two such signs were impressed. And twenty-four was written, as we do now, first the number of tens, and then the number of units. Like this:

"Yes, this is indeed simpler than hieroglyphs," Seva rejoiced.

"It’s not only simpler, but it’s already similar to our way of writing numbers. Units on the right, followed by tens, then hundreds… In short, all the numerals take their positions, as in formation. That’s why this method is called positional."

"So we record numbers using the positional method?" asked Tanya.

"Of course," replied Four. "And this began in Babylon."

"I understand," added Seva, "we have Babylonian counting…"

"That’s incorrect," Four stopped him. "Our counting is not Babylonian, but our own, special one. After all, we count in the decimal system, while the Babylonians had a sexagesimal one!"

"How is that?" asked Seva.

"Like this: let’s take some number, well, for example, 3662. In our system, the two here denotes the number of units, behind it stands the six — this is the number of tens, and the next six is the number of hundreds, finally, the three is the number of thousands.

So this number could be written as:

```
3000 + 600 + 60 + 2 = 3662.
```

But for Babylonians, everything was completely different. If they knew Arabic numerals, they would write this number as:

```
1 1 2.
```

In their system, the two, as in ours, remains the number of units — the first place value. But the one to the left of it is not the number of tens, but the number of sixties — the second place value. And the next one is already the number of 60 x 60 = 3600 — the third place value. Note that you must leave an empty space between place values, otherwise you can easily get confused, which, by the way, often happened.

Thus, our number in the Babylonian system would look like this:

```
3600 + 60 + 2 = 3662.
```

That’s how they counted," Four concluded.

"Oh, how difficult! It’s good that nobody counts like that now!" exclaimed Tanya.

"You’re mistaken," Four corrected her. "You also count like this… sometimes."

"Me? Never!"

"Let me remind you now. Tell me, please, how many minutes are in an hour?"

"Minutes? Sixty."

"Right. And how many seconds are in an hour?"

"I’ll tell you now. Sixty times sixty… Three thousand six hundred," Tanya calculated.

"You see. You divide hours and minutes not into ten parts, but into sixty! So you do count in sixties!"

Tanya just threw up her hands:

"I had no idea that we still have something from Ancient Babylon!"

Pushkin Museum

"We've been to so many places today!" Oleg said thoughtfully as we were returning to Arabella. "Rome, China, Egypt, ancient Slavs, Babylon, and yet we still haven't found little Zero anywhere."

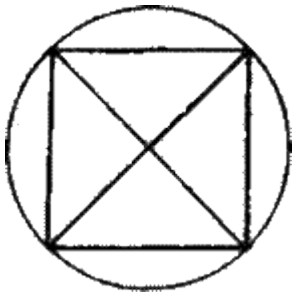
"It turns out that you and I were right," Four smiled slyly at me. "But don't worry, friends! We will definitely find little Zero! Just in case, let's check the Pushkin Museum."

"What, you have a Pushkin Museum?" the children were amazed. "A poet in the Arithmetic State? What connection does he have to you?"

"Pushkin was a very well-rounded person," Four objected. "He diligently studied history, loved music, and was interested in us, the residents of Arabella."

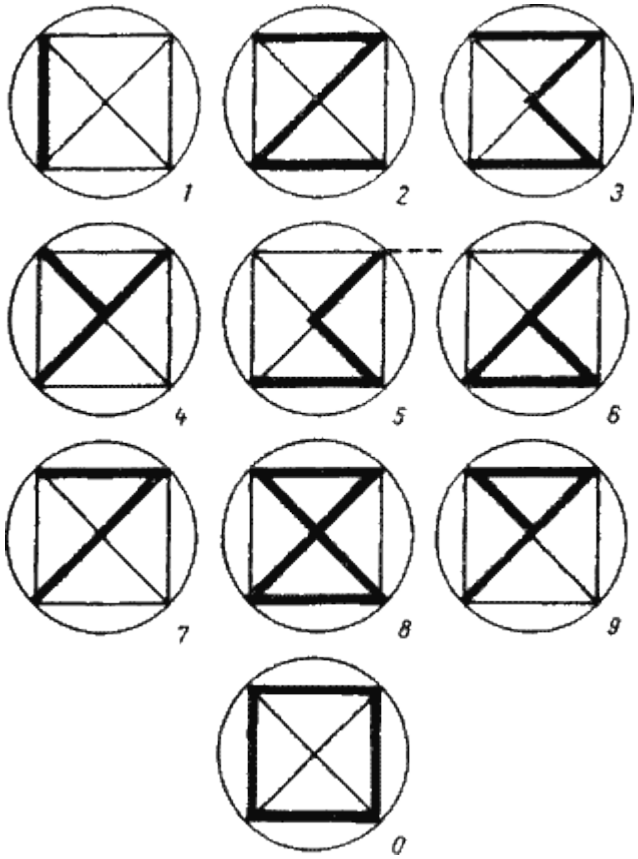
By this time, we approached a small house adorned with a portrait of the great poet.

Four with a bow led us into a room where there was nothing except a strange drawing hanging on the wall.



"This drawing was taken from Alexander Sergeyevich's manuscripts," Four continued. "The fact is that for a long time, people have been puzzling over where the symbols for Arabic numerals came from. There are many possible conjectures. Pushkin also proposed his own witty speculation, which we really liked. He decided that all ten Arabic numerals, including zero, fit into this magic square. To make it easier to understand his drawing, look here."

Four pulled out a large folder that we hadn't noticed at first. There were ten sheets inside. On each one was the same drawing, but each time a bold line outlined a new figure, in which we could easily recognize one of our numerals. Only the five was a bit off—it was missing its tail.



Four with a bow explained that in ancient times, the five didn’t have a tail. It grew somewhat later.

"Interesting!" said Oleg. "But can we consider Pushkin’s assumption to be correct?"

"Many dispute it. But we, the Arabellans, like it. It’s nice to think that you came from a magic square!"

"Even zero is square here," Seva chimed in.

"But our little Zero is still nowhere to be seen," Tanya sighed with distress. At that moment, we heard the chime of an old clock. It struck twelve.

"Oh dear!" Four hurried. "In an hour, a debate will begin, and I’m participating in it. We must hurry."

"What debate?" Seva inquired curiously.

"A very important debate at the Club for Debate Lovers. Announcements are hung all over the city. Haven’t you seen them?"

"We want to go too!" the children declared decisively.

"I would be very glad!" Four bowed courteously. "You can also take part in the debate."

"What’s the debate about?"

"About which is greater: 4/7 or 2/3. Apparently, not everyone here knows that yet."

And so we set off for the club.

Argument Enthusiasts

The hall was packed to capacity.

On the platform stood a large judge’s table and two small ones on the sides. To the right and left were platforms that resembled diving boards at a swimming pool.

A bell rang, and three judges in red robes ascended to the stage.

The Chief Judge raised a megaphone to his lips and began:

"Argument enthusiasts! We open our next, two million four hundred forty-first dispute. It was started yesterday by our junior schoolchildren. The argument that began in the classroom continued on the street. The opponents ended up with bruises and bumps. The teacher was unable to handle the fighters. And so we, argument enthusiasts, have received the pleasant opportunity to bring this dispute to our club. Long live the arguers! What would we do without them? Now, to the substance: some claim that the fraction 4/7 is greater than the fraction 2/3. Others, as you understand, prove the opposite. I ask the captains of both teams to take their places."

Two dwarf schoolgirls—One and Five—came up on stage. They sat down at the small tables.

The audience buzzed, whistled, and clapped.

"Don’t let us down, Five!" shouted some.

"Hang in there, One!" shouted others.

"Silence!" shouted the Chief Judge. The hall reluctantly grew quiet. "For complete clarity, I ask both fractions that caused the dispute to come up here."

Four dwarves, including our Four with a bow, took their places on the side platforms, forming the fractions:

4/7 and 2/3.

"The floor is given to One," thundered the megaphone.

One stood up, bowed to the judges, and began speaking:

"I assert with full responsibility that 4/7 is greater than 2/3." (Whistles, applause.) "No need to whistle! I have weighty evidence. Here it is."

One raised a stick above her head and waved it threateningly in the air. (Noise, excitement in the hall.) Then she approached the first fraction and placed the stick next to it.

"You see," said One, "this stick reaches Four right up to her bow. And now let’s measure the second fraction… Aha, what did I say? The stick is much higher than the upper digit 2!"

"That’s because I’m wearing slippers today!" Two squeaked, offended.

And again laughter, whistles, applause.

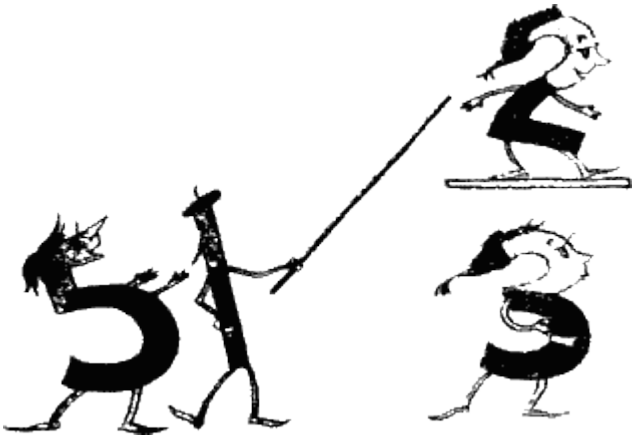
Having calmed the audience with difficulty, the Chief Judge gave the floor to Five.

"I don’t know what I should object to," she began calmly. "If my opponent is not laughing at us, then she is obviously stupid."

"I request that it be recorded in the minutes that I was insulted!" declared One.

"I call you to order, Five," said the Chief Judge.

"Great Judge," Five addressed him, "do we measure fractions with sticks? After all, some schoolchildren write large digits, others small ones. If we measure digits by height, then Nine might turn out to be smaller than little Zero."



"Ah, you don’t like my stick!" One jumped up from her seat. "You should have said so. I can give another proof. Let’s put both fractions on the scales. And you’ll see that the first weighs more than the second."

"Are you hinting," shouted the well-fed Seven, "that I ate too many meat pies for breakfast today?!" (Noise, laughter, applause) "I protest! Please note in the minutes that I was insulted."

"Quiet," said the Judge, "I don’t believe I gave you the floor!... Continue, Five."

"I have nothing to talk about," objected Five. "I know that numbers have weight, but this should be understood not in a literal sense, but in a figurative one."

"I object to this way of arguing," declared One. "Five dismisses all my evidence and doesn’t offer any herself. Because she doesn’t have any!" (Whistles, applause.)

"I can only repeat what I said at the beginning," Five replied calmly. "The value of a fraction is determined not by weight or height, but by its value!"

"Method, method!" One fumed. "You’re just babbling. You’re a show-off!"

"I request that it be noted in the minutes that I was insulted!" Five raised her voice.

"I give a stern warning to both arguers!" the Chief Judge roared again. "The dispute must be mutually polite. Continue."

"I assert that 2/3 is greater than 4/7," said Five. "And I’ll prove it to you now. Without sticks and scales! I’ll ask my assistants to come on stage. Two twins. Respected CD, please come up here with your brother."

Two identical numbers—21—appeared on stage.

"Why is she calling them CDs?" Seva whispered.

"Those are probably abbreviated names," said Oleg. "Of course, they’re Common Denominators—CDs!"

"These brothers," continued Five, "are nothing other than the product of the denominators of our fractions—Three and Seven. After all, seven multiplied by three equals twenty-one. I ask you, dear twins, to take the place of the denominators of both fractions: instead of Seven and Three."

"Respected Five," the Common Denominators objected in unison, "we cannot fulfill your request. If we now take the places of the denominators, you will lose the dispute—the first fraction will turn out to be less than the second!"

"Aha, what did I say?!" One rejoiced.

"Don’t rejoice prematurely," Five stopped her, "I just rushed a bit. Thank you, dear CDs, for your remark. Of course, we need to change the numerators of both fractions at the same time. I didn’t have time to say this. After all, when replacing denominators, the fractions themselves should not change. So, let’s replace both numerators and denominators simultaneously."

And then something extraordinary happened: Seven rose to Two, Three to Four, and between each pair, a multiplication sign instantly flashed.

The light went out for a second, and we saw new fractions on the sides of the stage: 14/21 and 12/21.

"Although these fractions are new," Five explained, "their values haven’t changed, have they? What do you think? 2/3=14/21, and 4/7=12/21. Right?" One made a contemptuous grimace and didn’t answer.

"So, my proof is ready! As you can see, the denominators of the fractions are the same, but the numerators are different. So which of these fractions is greater?"

"The one with the larger numerator!" Seva couldn’t help but exclaim.

"Please do not prompt from your seat!" thundered the Chief Judge.

"You are absolutely right, dear schoolboy," Five noted. "The fraction 14/21 is, of course, greater than the fraction 12/21. Therefore, truth is on my side."

The spectators applauded wildly. The judges, after conferring, stood up.

"I announce the court’s decision!" trumpeted the Chief Judge. "Five won!" ("Well done!" swept through the hall.) "From now on, I forbid using any other method when comparing fractions! The debate is over!"

"Attention!" someone shouted from the hall. "I have an announcement! A circus performance will be held today for the participants of the debate. An unprecedented stunt—'Fractions on Trapezes'! Entrance to the circus only with club tickets. The nervous are asked not to come."

The crowd poured into the street.

The Deadly Attraction

The orchestra played a cheerful introduction

At the ring, by the main entrance, the uniformed assistants lined up, and the performance began.

Jugglers were replaced by acrobats, acrobats by gymnasts... Then a slender, flexible Three ran into the arena; she performed a plastic study: first, to the music, she slowly transformed into a Six, then into a Nine, and finally into an Eight.

Then a young horsewoman — an elegant Five danced on the back of a horse, jumped at full speed through a hoop, and turned her little head so quickly to the right and left that no one could tell whether it was a Five or a Three.

Then a magician came to the ring. He rolled up his sleeves and asked each spectator to think of a number.

"Has everyone thought of one?" he asked.

"Everyone!" the spectators answered in chorus.

My companions also thought of a number — 11.

"Please," said the magician, "multiply your number by 6." The spectators began to multiply by six in their heads, moving their lips as they did so.

"Eleven times six," whispered Tanya, "equals sixty-six."

"Add 21 to the result," commanded the magician. "Have you added it?"

My kids got 87.

"Good!" came from the ring. "Divide the sum by 3" ("Twenty-nine!" Seva nudged me). Then subtract 5 ("That leaves twenty-four," the kids whispered). "Now divide by 2!" ordered the magician. "Have you divided?"

"Just a moment," someone shouted from the gallery. "One minute. Done!"

"It’s twelve," the kids exchanged glances.

"Now there’s only one thing left," concluded the magician, "subtract one. And I’ll tell you what answer each of you got. Everyone got the number they thought of. Right?"

"Right!" shouted Seva. "Eleven!"

"Right!" came from all sides. "Eight! Right — six! Right — five, seventeen, four!"

To thunderous applause, the magician bowed for a long time, and then moved on to the next trick.

"In this box are ordinary zeros. You know them well. I take this ax and chop each zero into any number of pieces." (The circus gasped in horror.) "This zero into five pieces, this one into seven, and this one into thirty-two. Done! Now inspect the box, it’s completely empty. I throw the fragments of zeros in here. I cover the box with a cloth. Attention!" The magician struck the box with his magic wand and pronounced: "Oy, lyuli, oy, lyuli! Come out, all you zeros!"

He quickly pulled off the cloth — zeros jumped out of the box one after another: they were completely whole!

The audience went wild.

"You see," said the magician, "no matter into how many parts I divide a zero, it always remains a zero. Zero divided by any number is zero! And now," he continued mysteriously, "**I** will show you

the most terrifying trick. I ask someone to come to the ring. Let it be the smallest dwarf, it doesn't matter. Before your eyes, I will divide him by zero! Who wants to come forward?"

No one appeared.

"Well," the magician shrugged, "I'll have to call my assistants."

He clapped his hands, and a fragile One in a pink tulle skirt and little Zero ran into the ring.

"So, I divide this One by Zero! Those who are easily frightened, please look away. Actually, I'd better cover them with this veil. Like this. And now I give them the division sign. Ready!"

And the magician pronounced a magic spell:

> Divide yourself by Zero quick,
> Appear before us with a trick!

Lightning flashed, a terrible thunder of drums sounded.

The veil quickly flew upward, and from under it emerged… a Giant!

He grew with incredible speed. Now his head was already touching the dome of the circus. Now it had broken through the canvas roof, and the Giant kept growing, growing…

The spectators huddled together in fear.

"Enough!" they shouted from their seats.

The magician waved his magic wand — and the Giant instantly disappeared. The fragile One and the little Zero were standing in the ring again.

"Now you are convinced," said the magician, "how dangerous it is to divide even One by Zero."

He elegantly bowed and left the ring to thunderous ovations.

"How does he do that?" asked Seva. "And where does the Giant come from?"

"That’s the point of magic tricks — not to understand right away," I replied. "However, I’ll explain this trick later. Now let’s watch the clowns."

Two clowns entered the arena from different sides: Tuk, white as flour, and Tok, orange as an apricot.

"Where have you been, Tok?" asked Tuk.

"I went to buy you a gift. Apricots!"

"I love apricots. Where are they?"

"I ate them on the way."

"All of them?"

"All of them. And then I went back to the store and asked them to sell me apricots again. But only half of what I bought the first time."

"Where are they?"

"Ate them!"

Tuk threateningly raised his stick.

"Wait, wait!" shouted Tok. "I went back to the store again and asked them to sell me only a quarter of the apricots that I bought the first time."

"And you ate them again?"

"Yes!... Then I went back and bought one-eighth. And ate them again." Tok laughed. "So I went back five times. Each time I bought half as much as the previous time. See, I didn't forget about you."

"Did you bring me any apricots or not?"

"Of course, I brought some. Here, look."

"But there's only one apricot!"

"I ran out of money."

"How many apricots did you eat altogether?"

"I didn't count. Count them yourself!"

"You ate them, and I have to count?"

"If you don't know arithmetic, let the audience help you."

"Friends," Tuk addressed the audience, "do you know how many apricots Tok ate?"

The circus became noisy, people started counting, arguing. Oleg was the first to solve the problem:

"Tok ate sixty-two apricots!"

"Wrong!" shouted Tok. "Sixty-three." He snatched the apricot from Tuk and immediately ate it. "This is the sixty-third!"

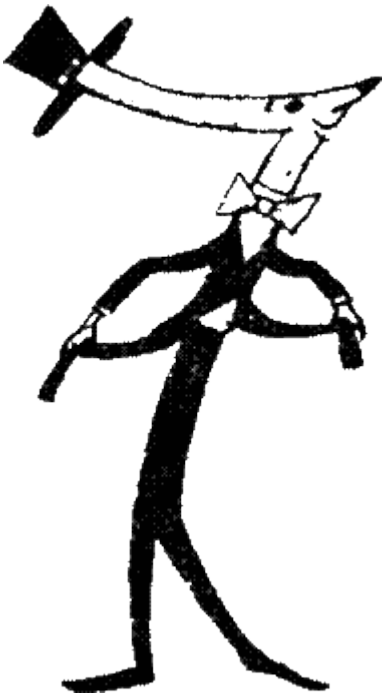
"Have you always been such a glutton?" asked Tuk.

"Always. Yesterday I ate twelve chocolate bars, fifteen pastries, and twenty portions of ice cream. That's how much!"

"And what was the sum total?"

"An upset stomach!"

In the end, the clowns began to solve a problem: how long would it take to fill a pool with water if it was filled through two pipes at once. Instead of pipes, Tuk and Tok had fire hoses in their hands. The clowns stumbled, fell, soaked themselves with water, and, without solving the problem, ran wet from the ring.



A dwarf in a tailcoat appeared from the main exit and solemnly announced:

"A deadly attraction! Fractions on trapezes!"

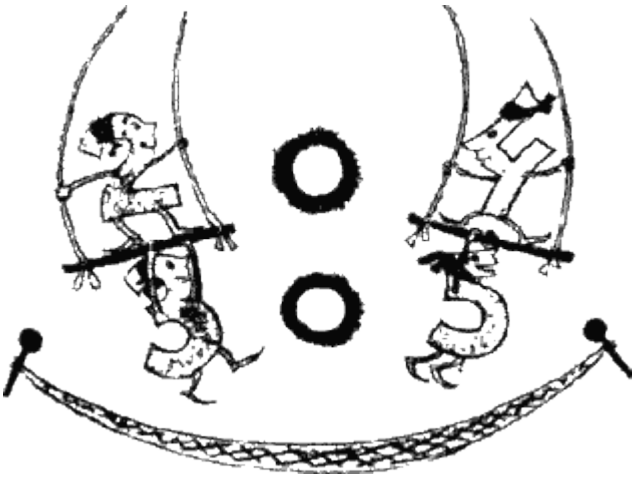
Two trapezes had already been lowered from the dome. They swayed opposite each other.

Four young dwarves ran into the ring: Two, Three, Four, and Five. They skillfully climbed up the rope to the trapezes: Two and Three to the left one, and Four and Five to the right one. Two and Four stood on the bars and firmly grabbed the ropes with their hands.

And the other two gymnasts hung on the bars below them. They swayed smoothly and made beautiful movements: arched like swallows, turned themselves upside down on their hands, hung head down… Then they sat on the bars to rest, and the dwarf in the tailcoat announced:

"Attention! The final trick — division of fractions!"

And immediately a drum roll ominously beat.



The gymnasts again took their original positions, and we understood that they represented two fractions. Between them appeared two glowing points — the division sign.

The gymnasts looked down — the safety net was in place. Without a net, this number was not allowed. A second of attention, and… "Up!" came the command.

And now Five, doing a somersault in the air, rose to the second trapeze and stood next to Two. Simultaneously, Four dove like a fish downward and grabbed the bar next to Three.

Again, as during the dispute, two multiplication signs flashed like lightning between the numbers. The light went out for a second, and when it came on, the fraction 10/12 was already swinging on the first trapeze. The circus roared with delight.

"Encore! Encore!…" came from all sides.

The trick was repeated. However, now the dividend and divisor exchanged places.

When the light came on again, a new fraction was swinging on the trapeze: 12/10.

"Hey!" shouted Seva. "It’s the same fraction, just upside down!"

"That’s what a circus is for," Oleg reasonably noted.

The artists easily descended to the ring. The show was over.

Found!

When we came out of the circus into the street, the city was festively illuminated. Despite the late hour, cheerful music could be heard everywhere.

The residents of Arabella were rejoicing. Here and there, exclamations could be heard:

— He’s found!

— What joy!

— He couldn’t have disappeared!

— Poor mother, how worried she was!

It’s not hard to guess who the dwarves were talking about. They were rejoicing over the return of little Zero.

This morning, when another group of dwarves was departing from the Square of Good Wishes to go to the humans, those seeing them off gave an order: to find Zero by all means. And now he’s here! He’s safe and sound!

And they found him like this.

About thirty dwarves were helping second-grade schoolchildren solve a problem on dividing whole numbers. All the students had already handed in their notebooks to the teacher. The bell rang. The dwarves left the classroom because a Russian language lesson was about to begin there, and they headed to the next class where they were studying common fractions.

They were climbing the stairs in a cheerful crowd. Suddenly, a pitiful squeak was heard on the landing. They looked around — no one! The squeak was repeated.

Then one clever Seven looked behind the trash bin standing in the corner. There she saw the missing Zero.

— I want to go to my mom! — he whimpered. — And then I want to eat and sleep. And I’m tired overall.

Well, they immediately comforted him, wiped his little nose and eyes. The only problem was they had nothing to feed him with yet. But Zero cheered up right away anyway, even jumped a little with joy. And everyone went to the third grade to help children add common fractions.

The problem was very difficult. And Zero had no time to tell what had happened to him. And afterward, they had to hurry to catch the plane. And only on the plane did Zero tell the sad tale of his disappearance.

He had to repeat this story many times: first to fellow passengers on the plane, then to those meeting them, then to those who were late for the meeting, and finally, to those who wanted to hear everything again from the very beginning. But since almost everyone wanted this, in total, Zero told his story 248 times.

And then we came up and also asked him to tell us. But Zero was already hoarse, and instead of words, he produced only hissing sounds.

And then the happy mother, the plump Eight, told everything for him. She had already managed to learn this story by heart.

— Yesterday, for the first time, I let my dear little one go on such a long journey. But I dared not hold him back. After all, he was going to you, humans, for a very important matter. And so they brought him to an incomplete middle school. I don’t know why it’s called incomplete, but the fact that it’s very middle is absolutely clear: that’s where the misfortune happened to my dear little one. And just imagine that my son had to end up with the worst student in the whole school. He was the one who lost my Zero. Yes, yes, lost! As if he were a needle! Is it conceivable to lose such a handsome boy! — She kissed Zero on his little nose. — The teacher assigned a very simple problem:

divide 1836 by 18. Is that difficult? Of course not! And the answer is very simple: 102. No more, no less. But this lazy boy got 12! Just 12! Just think — he lost Zero! And why did this happen? Because after dividing 18 by 18 and correctly getting 1, the student suddenly started dividing the number 36 by 18. He should have first divided 3 by 18, and only then 36. You'll say that 3 can't be divided by 18? Well, so what if it can't be divided! If a number can't be divided, that's exactly when my Zero should have been remembered and placed after the one. Then the correct answer — 102 — would have been obtained. But the boy was lazy and started dividing 36 by 18 right away. And so it turns out: haste makes waste! But we're not laughing, we're crying.

Well, okay. Let's assume the student made a mistake. Anyone can make mistakes. But he could have easily checked himself. You ask — how? He just had to multiply the quotient 12 by the divisor 18. And he would have gotten just 216 instead of the dividend, 1836. No, just think about it: to get 216 instead of 1836! Awful! Thanks to the kind friends — the dwarves. If not for them, my son would have been lost.

— Dear Eight, — Seva spoke up, — your Zero would have been found anyway. The teacher would have definitely found him. She simply hadn't had time to check the notebooks yet.

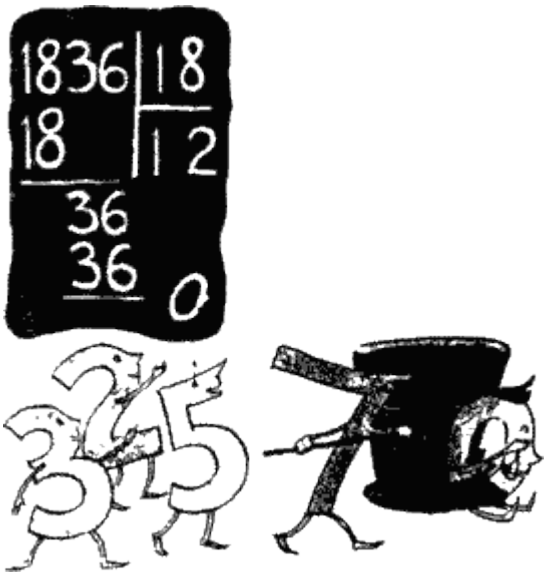
— Yes, — replied the plump Eight, — the teacher, of course, would have found him. But when? My Zero could have died of hunger by that time.

And she again began to hug her son, kiss him on his little nose, eyes, ears…

We decided not to distract her from this pleasant occupation and tactfully withdrew. But after a few steps, we again heard the familiar voice:

— Yesterday, for the first time, I let my dear little one on such a long…

The happy mother, at the request of the newly arrived, was beginning everything from the start.



Third Day

The Matchbox

"Remember," Seva said to me, "you promised yesterday to explain how the magician transformed One into a Giant."

"Well," I replied, "I promised, so I'll explain." The children sat closer and prepared to listen.

"Let's take some number," I began, "say, one hundred. And first, let's divide it by one hundred as well. We'll get one, won't we? Now, if we divide one hundred by fifty, what do we get then?"

"Two!"

"Correct, two. Two is already greater than one. And if we divide one hundred by twenty-five, we get even more—four. Then by twenty. The quotient will be five. And if we divide one hundred by two, the quotient will be even larger—fifty. Right? It turns out that the smaller the divisor, the larger the quotient. Now let's divide one hundred by one."

"It remains one hundred," said Seva.

"That wasn't hard to figure out," I continued. "But what if we start dividing one hundred by numbers smaller than one? What then? Will the quotient decrease or increase even more?"

"Increase," said Tanya.

"Of course. The smaller the divisor, the larger and larger the quotient. Divide 100 by 1/2, and you get 200, and if you divide 100 by 1/5, the quotient will be 500."

"Of course," said Oleg, "dividing by 1/5 is the same as multiplying by 5."

"Well done," I praised Oleg. "So, if we divide a number by one millionth, then..."

"...it's the same as multiplying that number by a million," Seva triumphantly concluded.

"Now think about this," I said again, "is zero a small number or a large one?"

"Zero is smaller than any small number," replied Oleg.

"What happens if you divide one hundred by the smallest number?" I asked again.

"The same thing that happens if you multiply one hundred by the largest number," answered Seva.

"Correct," I confirmed. "The magician divided one by zero—and a Giant appeared! No tricks at all!"

The children sighed with satisfaction.

"Now I'll show you a real trick!" I continued after a brief pause. "How many numbers do you think can fit in this matchbox?"

"It depends on how you write them," Seva said thoughtfully, "large or small."

"Let's say small," I decided generously.

"Then—many," replied Tanya.

"What does 'many' mean?"

"A thousand!" exclaimed Seva.

"More."

"A million!" Tanya suggested uncertainly.

"Even more!" I encouraged.

"Now that's just a fairy tale!" Seva grumbled skeptically.

"Well, listen to my fairy tale. A tale that's not quite a tale." I took all the matches out of the box. "Let's say this box is divided into two equal parts, say, with a match. Let's place the number 1 in one part."

"Write down the one," Seva proposed businesslike, holding out a pencil.

"No," I objected. "The one will be imaginary. We mathematicians can't do without imagination! So, in this half—there's a one, and the other half is empty."

"Very uneconomical," declared Seva. "Taking up half a box with just one number."

"No matter," I replied, "there's plenty of space. Now let's divide the free half in half again. In our imagination, of course. Can we do that?"

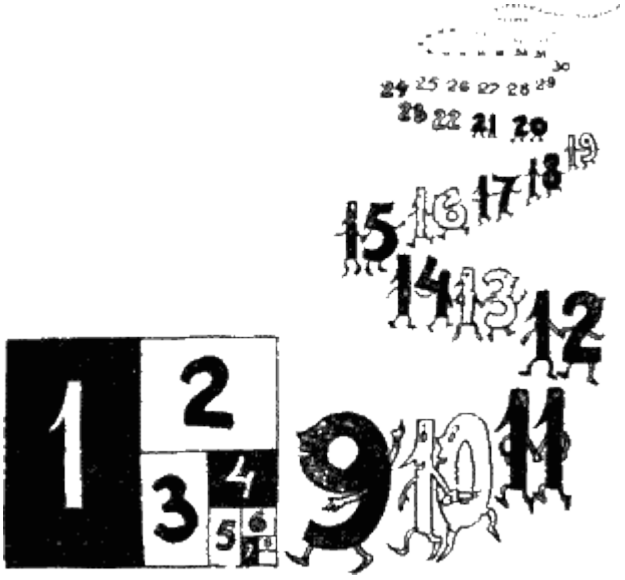
"We can!" the children said.

So, we have two empty compartments again. In one of them, we'll mentally place the number 2. And we'll divide the free compartment in half once more. And in one of these halves, we'll place the number 3. Then we'll do the same thing again. We'll continue each time placing a number in one of the free compartments: 4, then 5, 6, 7... 100... 1000, and so on. And each time we'll divide the free compartment in half again.

"No," Seva stopped me, "something's not right here. How will you keep dividing the box? If you use matches, they won't fit."

"I'll use hairs instead of matches," I answered.

"It still won't work," Seva persisted. "You might divide the box into five hundred, a thousand parts, but after that, even a hair won't fit!"



"What poor imagination you have!" I shook my head. "Didn't the blacksmith Lefty manage to shoe a flea and even sign his name on each nail? Not so long ago, there was no smaller division of time than a second. But now scientists have learned to measure even billionths of a second! In the old days, to praise a spinner, they would say she spun thread as thin as a cobweb. They couldn't imagine anything thinner than a cobweb. And they certainly couldn't measure a cobweb. But now they measure the sizes of molecules, atoms, electrons... Compared to these, a cobweb is like an oak tree next to a midge! So. Let's say there's such a skilled craftsman who can divide our box into the very smallest compartments. We don't need to look far: isn't imagination the best craftsman in the world? So, the craftsman works, the compartments become smaller and smaller, now you can't see them even with a microscope! But the craftsman keeps dividing and dividing. The compartments become ever smaller, and the numbers placed in them—ever larger. And the smaller the compartment, the larger the number we place in it. Will there be an end to this? No, there won't! After all, we can divide without end, and there are infinitely many large numbers too. So it turns out that in this box, all infinitely small and all infinitely large values have gathered. Dwarfs and giants!"

"So that's why this country is called Dwarfland!" Oleg exclaimed happily.

"Oleg the Seer!" said Tanya.

The Little Zeros Are Up to Mischief Again

The fairy tale made a great impression. The children couldn't calm down, endlessly discussing the unusual "trick."

Fortunately, their chattering was interrupted by Four with a bow, who came running in out of breath. She came to tell us that she couldn't accompany us right now: the little Zeros were misbehaving so much that they couldn't be managed. And today was her duty at Number Square. She immediately rushed off. We hurried after her and this is what we saw. All sorts of numbers were strolling around Number Square, arm in arm. Two and Three were whispering about something, forming the number 23. Nearby, the first six digits marched in a row singing a song. They formed a large number — 123456, one hundred twenty-three thousand four hundred fifty-six...

Among these solid, dignified numbers scurried the mischievous little Zeros who had run away from their mothers from Decimal Lane. One of them, particularly bold, ran up to the number 125 and stood to the left of One, like this: 0125. No one paid much attention to him because the number 125 didn’t change a bit from this. Then the little Zero ran to the other end of the number and stood next to Five. The number instantly grew as if on yeast, becoming ten times larger: not 125, but 1250!

Since all the digits in this number were young ladies, they didn’t want to turn into old women at all. They chased the little Zero away.

Then the little Zero came up with a new prank: he again ran to the left of One and separated himself from the number 125 with a decimal point! And so it turned into a decimal fraction: 0.125 — becoming a thousand times smaller than it had been until now.

The digits were outraged:

"How dare you make such a small number out of us! We don’t want to shrink for no reason at all!"

But the little Zero liked this so much that he called two more of his friends and squeezed them in between One and the decimal point.

Oh no! Now the number 125 decreased a hundred thousand times and became this tiny: 0.00125!

Meanwhile, the little Zero developed a taste for this fun game. He was reveling in his power.

"Just think," he exclaimed joyfully, "it turns out that the more Zeros stand right after the decimal point, the smaller the number becomes! After all, each of us reduces it by ten times!"

As soon as he said this, five more Zeros pushed One aside and stood between her and their brothers.

"Now you’re no longer one hundred twenty-five," shouted the Zeros, "but one hundred twenty-five ten-billionths! Look: 0.0000000125!"

Oh, horror! The number became so small that you couldn’t see it without a microscope.

What’s the point of talking to an invisible entity!

The Zeros disappointedly left their places, and — ha-ha! — the number 125 was once again calmly walking around the square as if nothing had happened.

"Just you wait," Four with a bow said militantly, "I’ll teach you a lesson now! You’re taking pride in your power for nothing," she addressed the Zeros. "Look, the number 9.1 is walking around. Stand between the decimal point and One. Let’s see if the number will decrease by much?"

"Ho-ho-ho!" the little Zero responded defiantly. "Of course, by a lot!" Three Zeros quickly positioned themselves as suggested, and what happened? The number 9.1 turned into merely 9.0001.

The Zeros even began to whine:

"This is cheating! The number should have decreased a thousand times!"

"You didn’t consider that there’s a nine before the decimal point, not a zero! In these cases, your

power isn’t very great. You can put a hundred Zeros after the decimal point, and the number will still be greater than nine! So there’s nothing to boast about!"

But it was difficult to calm the little Zeros down. For a long time, they continued to annoy the numbers with their pranks, and finally, they became so tiresome that everyone’s patience ran out.

They had to call the manager of the main warehouse. He came, finishing his nine hundred and eighty-first ham sandwich, quickly and efficiently gathered all the Zeros, and placed a plus sign between them. All the Zeros immediately turned into one common Zero. No matter how many Zeros you add together, they will still not be more than zero!

The common Zero rolled across the square, rolled into its Decimal Lane, and there it hit a pole and broke up again into many small Zeros. Then their mothers caught them and took them to their homes.

Mirror Street

Everything quieted down in the square. Four with a bow came up to us.

"I have a surprise for you," she said, smiling mysteriously.

"What? What?" the children pestered her.

"What kind of surprise would it be if I blabbed about it ahead of time!" Four defended herself. "First, I’ll lead you by the nose a little bit."

"How long will you lead us?" asked the impatient Seva.

"I said just a little bit. We’ll walk down this street, turn into an alley, then go back to the street, turn into another alley..."

"Ugh!" Tanya groaned in disappointment. "So far?"

"Well, well, I was joking. We won’t have to go anywhere. Look over here."

We turned around and... were stunned. Before us stood a brand new, fresh-off-the-assembly-line bus! It gleamed in the sun with its red glossy sides.

"Is this really for us?!" exclaimed Seva.

"Of course!" Four replied, beaming with joy. "This is a long-distance tour bus. We’ll take a trip on it down one remarkable street."

Tanya pouted and said in a disgruntled tone:

"Was it worth bringing a long-distance bus just to drive down one street!"

"Not at all!" Four objected. "We won’t be able to drive to the end of this street."

"And why is that?"

"Because that street has no end!"

"I understand," said Oleg. "This road leads to giants."

"Not only to giants but also to dwarves," added Four.

"But we were just told about this today!" Tanya exclaimed, throwing up her hands.

"All the better," Four rejoiced and, with the gesture of an experienced guide, invited us to take our seats on the bus.

The children happily settled into the soft, comfortable seats. Four turned some lever, and the bus started moving.

"But where's the driver?" the children wondered. "Is the machine driving itself?"

"The driver is far away, on Automatic Street. He controls the bus remotely."

"This is probably done by a cybernetic machine?" Oleg suggested.

"Of course!" Four said proudly. "In Arabella, everything is state-of-the-art technology! Now, attention! We're approaching the destination of our journey."

The bus drove onto a wide street. Now it was moving very slowly.

"Look to the left," said Four.

We turned our heads and saw a long, infinitely long brick wall on which decimal fractions were lined up:

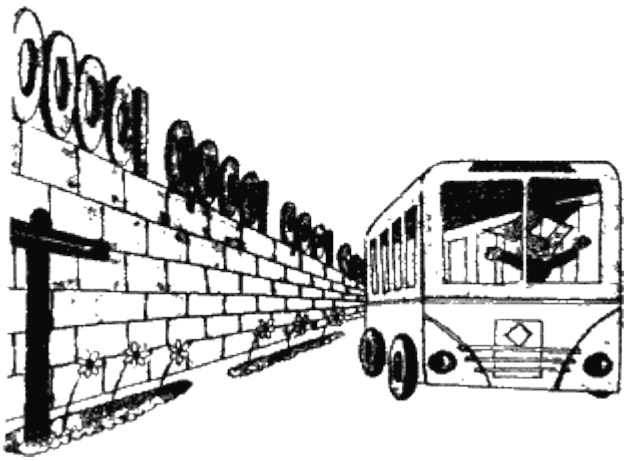
0.1 0.01 0.001 0.0001 0.00001 0.000001 and so on...

The further we went, the more zeros stood after the decimal point, and consequently, the smaller the fraction became.

We drove on and on, and the numbers became smaller and smaller. The bus gradually picked up speed. The zeros flew past us faster and faster. There were more and more of them. Numbers appeared that were so tiny that there was no way to read them. And the street had no end!

"This truly is the road of dwarves!" exclaimed Oleg. I had never seen him so animated.

"The dwarves are here," confirmed Seva. "But where are the promised giants?"



"There will be giants too," Four reassured. "Hold on tight! Everyone please close your eyes. I'm setting cosmic speed!"

Of course, no one was planning to close their eyes, but suddenly the bus reversed and shot back so rapidly that everyone involuntarily squeezed their eyes shut.

Before we could, as they say, blink an eye, we found ourselves back at our starting point. The bus stopped.

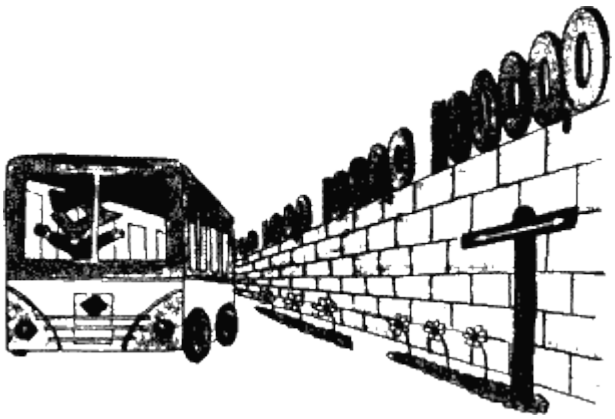
"Now look to the right!" Four commanded. We turned our heads and again saw a brick wall. But, strangely enough, there was a bus standing by the wall that looked exactly like ours.

"Look, another tour!" shouted Seva and waved his hand.

Someone from the bus waved back. Seva waved again. The bus responded again.

"Listen!" Oleg shouted in extreme excitement. "This isn't a wall — it's a mirror!"

Indeed, it was an infinitely long mirror, and the brick wall was reflected in it.



Numbers still stretched into the infinite distance. But now they weren't decimal fractions, but their reflections, transformed into whole numbers: 1.0 10.0 100.0 1000.0 10000.0 100000.0 and so on.

The bus started moving, and again zeros flashed before our eyes, zeros, zeros... The numbers grew with incredible speed.

"There's your road of giants!" said Four.

And then a fun game began. The children would turn left — before them were dwarves; they would turn right — giants would grow.

"Just like in a fairy tale!" Tanya exclaimed in delight. By the end, everyone's necks were aching. This was the signal to return.

"Just think," said Oleg on the way back, "from the same digits, you can make both dwarves and giants! It all depends on which side you look at them from."

Underground Crusher

We parted with our nice guide and, after resting from the strong impressions, went to wander

around the city again. Soon, a distant rumble was heard.

"Is it a thunderstorm?" Tanya trembled. She was afraid of thunderstorms.

"No," said Oleg, "it’s something else."

"Let’s go and see," Seva suggested. And we followed the noise.

It was getting louder and finally led us to some deserted alley.

Suddenly the rumble stopped. And it became so quiet that everyone got scared.

And then we heard someone’s creaky, grumbling voice.

From underground, the gray head of an old dwarf appeared. He was breathing heavily and, after scrambling up, immediately sat down to rest.

"Where are you from?" asked Tanya.

"From down there," he pointed downward.

"What are you doing there?"

"Working. It’s a hellish job I have."

"Is that the dwarven hell?" asked Seva.

"What does hell have to do with it?!" the old man was surprised. "Even our children know there’s no hell. I turn the machine there. I don’t have any more strength. Turn, turn, and all for nothing. Let the elder dwarves try themselves. And it’s time for me to retire!"

"And what kind of machine do you turn, grandpa?" asked Tanya.

"Don’t even ask. The most useless machine. And it’s called a crusher. But what’s the use of this crusher if I can’t crush anything completely?"

"What are you crushing?" asked Seva.

"What else besides numbers?"

"Numbers aren’t stones, is it really so hard to crush them?" Tanya asked again.

"Try it yourself once, then you won’t ask such questions. After all, I crush numbers that don’t divide into each other. Those that divide themselves — there’s no need to crush them. But you try to divide a number that doesn’t want to divide by another."

"In that case, you have to leave these two numbers in the form of a common fraction," Tanya advised.

"Look how quick she is!" the old man got angry. "In the form of a common fraction!" This is the decimal fraction quarter. Common ones aren’t allowed to live here."

"So you transform them into decimals?" Tanya persisted.

"Exactly — into decimals!" the old man waved his hand. "And they resist. Understand?"

"What kind of fractions are these?"

"Strange question! Periodic, of course!" replied the dwarf.

"But what's the use of them if not a single one can be crushed completely?" asked Seva.

"There is, of course, some use in them," the old man scratched behind his ear. "A periodic fraction, in general, comes very close to a common one. The more I crush, the more accurate the answer. Just keep turning! And I'm old, it's hard for me."

"Can we see your crusher?" asked Seva.

"Why not? Everything is allowed here. You can even crush on it if you want. And I'll rest a bit."

"Of course, with pleasure!" everyone declared in unison.

It turned out that the dwarf didn't grow from under the ground at all, but came up from a round hatch, which we hadn't noticed at first. Down a narrow spiral staircase, we descended into the dungeon, where next to the crusher stood a roughly built table. On the table lay a paper with a seal. It was an order from the Council of Elders:

"We propose to divide one by three using decimal fractions within 24 hours. Present in the form of a periodic fraction with accuracy up to a million digits. The Elders."

Nine signatures stood below.

"You see," said the old man, "with accuracy up to a million digits. They should try to do it themselves!"

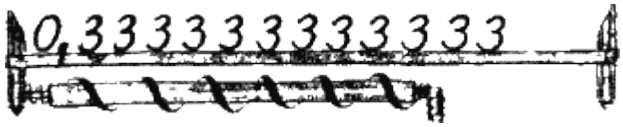
"Let me try!" declared Seva and began to turn the handle of the huge crusher.

Digits started jumping out of the crusher and lining up on the shelf. First jumped out zero, followed by a comma. And then came threes, threes... one after another. Seva was already tired, but the threes kept coming.

A very long queue had already formed on the shelf:

```
0,333333333333333333333333333333333333⋯
```

And the threes continued to jump out.



"Perhaps enough?" asked Seva, wiping sweat from his face. "Not a million digits yet?"

"Far from a million," the old man chuckled. "But it’s unnecessary here. The Elders made a slight mistake. They’ve already checked this crushing. They know in advance that nothing but threes will jump out of the crusher. I’ve sent them the answer more than once."

"And what, did you really send a million digits?" Tanya was surprised.

"Why?" the old man waved his hands. "There’s no need to send a million. I came up with sending them a shortened answer. Here, read it."

Seva took the paper and read:

"In response to your order, I report: when dividing one by three, this is what happened: 0,(3). Senior Crusher."

"And what are these brackets?" asked Seva.

"This is how we agreed to write that, well, except for threes, you shouldn’t expect anything more. And we called three the period. That’s why the fraction is called periodic. Here’s another order," continued the Senior Crusher, "to divide one by seven. This will have to be done."

"Now I’ll do the crushing," declared Oleg.

The crusher went to work again. Once more, zero jumped out first, followed by a comma. Then one

appeared.

"Well, now only ones will come," said Seva.

"Don’t get ahead," the old man stopped him.

And he was right. After one jumped out four, then two, then eight, five, seven:

```
0,142857.
```

"This is no longer a periodic fraction!" Seva rejoiced.

"Don’t listen to him," the Crusher addressed Oleg. "Just keep working."

Oleg continued to turn the handle, and… one jumped out again. And after it again four, two, eight, five, and finally, seven! They lined up in a row:

```
0,142857142857.
```

"So everything will start from the beginning again," decided Oleg. "Obviously, these six digits will now be called the period? Look, what a big period!"

"And I know how to write this result!" Seva rejoiced. "Like this: 0,(142857). Did I put the period in brackets correctly?"

"Couldn’t be better," confirmed the old man.

"But what if something else comes out further?" Tanya didn’t believe. "Let me turn it some more."

But no matter how much the children turned, the same digits kept jumping out and always in the same order.

When we climbed back to the street, a long queue had lined up on the shelf again:

```
0,142857142857142857142857142857142857142857142857⋯
```

We got out of the hatch and looked down. The Senior Crusher was already dividing a new fraction. We only saw his notation:

```
4/11 = 0,(36)
```

"We must check all this at home," said Oleg. Here, Seva was suddenly struck with an idea.

"Grandpa!" he shouted into the hatch. "Grandpa! Why don’t you retire? Let some calculating machine do your work instead. And you can rest as much as you like!"

"Me, retire?" the old dwarf raged. "I’ve been crushing for a whole century, and now I’m not needed

anymore?"

The children looked at each other in embarrassment.

"But you yourself said…" Tanya timidly reminded.

"So what if I said! They've been trying to convince me to rest for twenty years now, but they picked the wrong person! I can't live a day without this work. 'Retire'! I'll show you!… Get out of here, or I'll turn you all into periodic fractions! You'll remember me!"

So unexpectedly ended our visit to the underground crusher.

O.R.C.C.

This day — our last day in Dwarfland — was truly full of surprises.

The good fairy — Four with a bow — found us again to deliver an invitation from the Council of Elders to a ceremonial reception.

The children were a bit confused — they had never been to receptions before, especially ceremonial ones.

"What do you think," Tanya asked with concern, "is my dress suitable for such an occasion?"

"It is," I answered confidently. "In Dwarfland, fanciness is not particularly encouraged, but cleanliness is valued in the highest degree."

Tanya was pleased to note that the pleats of her school skirt were well-ironed, and her festive apron shone with whiteness.

Oleg's outfit was also in order. With Seva, the situation was somewhat more complicated: two buttons were missing from his jacket, and his pockets bulged with all sorts of things. "Just like Tom Sawyer," Tanya would say about him. However, after some adjustment, Seva acquired an almost tolerable appearance. And all of us, anxious and timid, headed to the Number Square. An enormous round ten-story building seemed to consist entirely of glass and could be seen through like a giant lantern.

Each of the nine rulers of Dwarfland occupied one floor. The top, tenth floor was common. There, in a round hall called the Hall of Knowledge, the Council of Elders gathered. From there, one could see not only the streets, squares, lanes, and dead ends of Arabella, but also the endless fields, forests, plains, and mountains of the Arithmetic State — all its remote corners.

Four worn steps led to a wide, massive door. On each of them was written one letter.

If read from the bottom, they spelled: O.R.C.C.

Seva had already raised his foot to step onto the first step, but Four with a bow managed to stop him.

"No, no!" she exclaimed. "You are not yet allowed to climb these steps. There is a separate entrance for guests of your age."

"But what do these letters mean?" asked Seva when we were climbing the side staircase.

"These are the initial letters of four words — our main motto. These are the most important, most necessary, most beloved, most great words in the entire Arithmetic State."

"But what are these words?" Seva insisted.

Four didn't have time to answer. The wide door opened, and we saw a majestic scene. It's useless to describe it. Let everyone imagine it in their own way.

Upon our appearance, the nine Elders stood up, greeted us, and sat down again only when we took our assigned places. In this hall, they did not waste words or time. Here, they never talked about trifles, gossiped, or spoke ill of others. Every second was valued for its weight in wisdom, and that is more precious than gold.

"Dear guests," began the first Elder, "we have invited you to the Hall of Knowledge to broaden your horizons a little. Through the glass walls, you can see the boundless distances. These distances have no limits. This is the essence of knowledge. Any knowledge, even small, does not come easily. Four steps — O.R.C.C. — lead to this Hall: *Observation, Reflection, Calculation, Conclusions!*

For hundreds of centuries, many have climbed these steps, entered the Hall of Knowledge to then tell others about what they saw and introduce them to science. Many stumbled on these steps and did not rise above the first or second.

There were also those who tried to enter the Hall of Knowledge through bypasses. But this led to nothing. For such people, the walls of our Hall become opaque. The deceivers themselves become deceived.

They say that there is nothing interesting in knowledge, and they go back, confusing with their stories those who want to enter our Hall through honest means. Fortunately, there are far fewer such ignoramuses than good, inquisitive people. It's no wonder that the four steps leading to the heights of science are so worn.

Today, we welcome you as our dear guests. But we believe: the time will come — you will climb these four steps and enter here not as guests, but as masters."

The first Elder finished. A short pause followed.

Then the second Elder rose.

Here is his story.

Space in the Room

— I will tell you about a man who lived about a hundred years ago. He slowly but persistently climbed four steep steps and entered the Hall of Knowledge as a victor.

I want to tell you about him because this man elevated not only himself, not only humanity, but also Dwarfland. Numbers played a very important role in his work.

This man's name was Urbain Jean Joseph Le Verrier. He was French.

Le Verrier loved stars, planets, and, of course, our Earth very much. And especially he loved us, the dwarves. That's why he became a great mathematician.

Dividing and multiplying huge numbers was as simple for him as playing volleyball is for you. He dealt with numbers more easily than a juggler with balls.

Day and night he sat at his desk calculating, calculating, and then he would go to the telescope and look at the stars. He could observe them endlessly.

As you can see, it all started with observation! This is the first word of our motto. After all, if you don't observe, you won't notice anything, and then there will be nothing to think about!

Le Verrier observed the planets, their movement. He was particularly interested in the curious character of one of them, called "Uranus."

You, of course, know that all planets move around the Sun, each in its own orbit, just like motorcyclists during a race in a stadium.

They behave very disciplined — not one deviates from its track. Neither Mercury, the planet closest to the Sun, nor Venus, running on the second track, nor the next on the third track — our Earth. All race along their orbits — Mars, Jupiter, Saturn, and finally, Uranus.

No other planets were known in Le Verrier's time. Le Verrier himself didn't know about them either. But here's what surprised him. All planets move along the orbits calculated for them, all obey celestial laws, but Uranus doesn't want to obey them. It runs part of its track and then turns to the neighboring one, even further away from the Sun. Fortunately, this track is empty — after all, Uranus is the last planet, and there's no one to collide with there. Le Verrier carefully studied the records made by various scientists before him. Everyone noted the strange behavior of Uranus.

And so Le Verrier began to wonder: why doesn't Uranus like to run on its track? Why does it tend to turn away from it?

This is how boys usually behave when they hurry to school. They run, run down the street, and suddenly turn off the road into some alley. And there, it turns out, they sell ice cream. The boy buys ice cream and runs back out to the main street. Tasty ice cream attracts boys — that's why they deviate from their path.

"Maybe something is attracting Uranus too," decided Le Verrier, "and it's running for its ice cream?"

You know that all planets and the Sun experience mutual attraction. If it weren't for the force of solar attraction, our Earth would have long flown away from the Sun, as they say, to the ends of the earth.

And the larger the planet, the stronger its attraction. The Sun is significantly larger than all its planets combined, so it doesn't let them run away.

But, of course, the further the planet, the harder it is for the Sun to attract it. And Uranus, as you already know, is the most distant planet from the Sun.

Le Verrier thought and thought and decided this: "Could Uranus be attracted by some other, unknown planet, even more distant from the Sun? Even if no one knows anything about this planet, it still must exist."

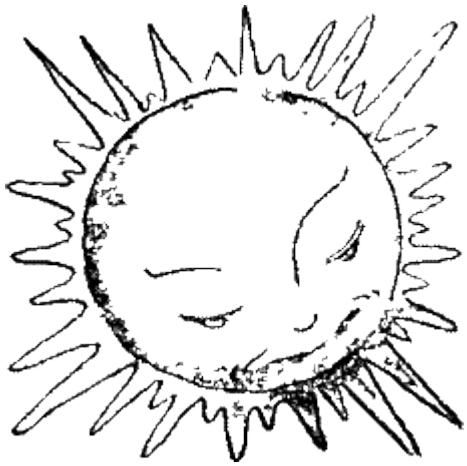
"This mysterious planet," reasoned the scientist, "attracts Uranus when it passes by, racing along its track. They draw level, like two racers. The attraction of the unknown planet becomes stronger than the solar one for this time. That's when Uranus turns from its path. But only temporarily. Because each planet moves at its own speed. Once Uranus surges ahead, the attraction force of the unknown planet weakens, and the runaway Uranus returns to its track again."

When Le Verrier finished his reasoning, he began to calculate. It's not enough to imagine a new planet; you have to prove that it exists. This is where we, the dwarves, become indispensable!

The scientist worked for a long time.

We were all very worried: would he be able to correctly calculate where and how to find the new planet? After all, in those days there were no calculating machines yet. The scientist filled mountains of paper. He was very tired, but the desire to achieve his goal and faith in his righteousness won.

Le Verrier completed his calculations. Now he knew where and when to look for the unknown planet; he even calculated how much it weighs and how far it is from the Sun, how long its orbit is and how long it takes to run once around the Sun. It turned out that one year is not enough for this. It needs almost one hundred and sixty-five years to make one turn around the Sun.



The scientist learned all this so precisely as if he had seen it with his own eyes. In reality, he never left his study.

When Le Verrier finished calculating, he sent his conclusions to the observatory, from where astronomers observe the stars.

They pointed the telescope at that place in the sky where Le Verrier directed, and at exactly the appointed time they saw a new luminous point. This was the planet calculated by Le Verrier.

So we learned that not seven, but eight planets race around the Sun in their orbits.

Scientists named the eighth planet Neptune.

And a few decades later, everything repeated from the beginning.

The English scientist Percival Lowell also calculated a new planet, the ninth in order, located even further than Neptune. Only many years later did astronomers find it in the sky.

This planet was named Pluto, and it needs almost two hundred and fifty years to run around the Sun once.

Maybe there is another, tenth, planet beyond Pluto? Who knows! It is quite possible that the honor of its discovery will fall to one of you…

The Second Elder bowed and sat down.

Then the third stood up.

"Look here," he said, pointing to the glass wall. "Before you is the path of the Bright Mind. Here are portraits of those who managed to climb the four steps to the Hall of Knowledge."

The Third Elder pressed a button.

The walls of the Hall of Knowledge began to move apart and simultaneously retreat. The Hall moved away more and more. Now the Elders are somewhere very far away, now they turned into barely visible points, and finally Dwarfland disappeared completely.

The Road of Bright Reason

Before us was a brightly lit endless road of Bright Reason.

To the right and left, long lines of portraits of great mathematicians stretched into the distance. Who wasn’t there! We saw the intelligent, penetrating eyes of those who had built the magnificent Arithmetic state brick by brick.

We walked among them, and it seemed to us that the portraits were smiling, about to speak at any

moment.

But what’s this? They are indeed smiling. They are actually talking, reaching out their hands to us. The portraits are alive!

Now someone is stepping out of a tight frame and walking toward us. He has a gray beard, and his curly hair is tied with a ribbon.

"It’s Archimedes!" Oleg recognizes him.

"Strange," says Tanya, "didn’t he die?"

"Yes, it was more than two thousand years ago," confirms Oleg.

"You are mistaken," Archimedes answers, smiling into his beard, "I did not die. You probably mean that sad incident when a contemptible Roman soldier pierced me with a spear. He also thought I died, but he was gravely mistaken. Unfortunately, he only interrupted me from solving a problem that I had drawn in the sand. I warned him: 'Do not touch my figures!' But he was deaf to science. Do you know the name of this vile warrior?"

"I haven’t the slightest idea!" replied Seva.

"Well, you see, I don’t know his name either."

"But people know Archimedes' laws very well," I said.

"I’m glad to hear it," Archimedes bowed. "However, what I discovered are not my laws. They are the great laws of nature. They existed long before me. Always! I just managed to glimpse them."

At that moment, Tanya blinked rapidly and, taking out a handkerchief, began carefully rubbing her eye.

"Why are you crying, dear girl?" Archimedes asked sympathetically. "Have I upset you somehow?"

"No, not at all!" replied Tanya. "I got a grain of sand in my eye."

"Just a grain of sand! Trifles!" Seva said dismissively.

"Trifles?" Archimedes took offense. "Young man, never speak without thinking. I devoted several years of my life to grains of sand."

"Ordinary grains of sand?" Seva was surprised.

"The most ordinary ones. I set out to calculate how many grains of sand could fit in the Universe if it were completely filled with sand—regular sand, like what you find on a beach."

"Is it possible to count that?" Seva threw up his hands. "Probably an infinitely large number of sand grains would fit in the Universe!"

"No, no, you’re expressing yourself incorrectly!" the ancient scientist stopped him. "You mean to say: not an infinitely large number, but a very large number. These are completely different things."

"But the Universe is infinite!" Tanya joined the dispute.

"You forget," Archimedes calmly replied, "that we, the ancients, imagined the Universe differently. In my time, it was believed that the immobile Earth was at the center of the Universe, and around it revolved the Sun, planets, and all stars, attached to the firmament like to the lid of an enormous bowl. So, I did count this huge number of sand grains and even wrote a treatise on the subject. I called it: 'On the Counting of Sand in the Sphere of Fixed Stars.' I would gladly give you this treatise with my autograph, but—alas!—I don't have a copy at hand. If you find one somewhere, I would be delighted to sign it."

"That would be wonderful!" Tanya lit up. "I love collecting autographs! I already have Yuri Gagarin's autograph, plus two movie actors. But Archimedes' autograph!..." Tanya even closed her eyes with pleasure.

"I see that the sand grain has jumped out of your eye on its own," said Archimedes, "I'm very glad... And now please excuse me—I have business to attend to. Perhaps today I will finally find the point I've been looking for so long."

"What point?" asked Seva.

"The fulcrum, of course. After all, if I find a fulcrum, I can move the Earth."

"How?"

"With a lever. Once, when I lived in my hometown—Syracuse, I invented this very simple machine..."

Waving goodbye to us, Archimedes quickly departed.

And we continued on our way.

Along the road, we saw those already familiar to us: Eratosthenes with his famous sieve, and Leverrier busy with his latest calculation, and Euclid, and Pythagoras...

And here come two men in old-fashioned doublets. They are animatedly discussing something. Now they have drawn level with us.

"Allow me to introduce myself," one of them addressed us, with long fair hair. "Isaac Newton. From Cambridge. And this," he pointed to his companion, "is Gottfried Wilhelm Leibniz. From Leipzig. We lived and worked in different countries, but always respected each other."

"Oh yes!" confirmed Leibniz. "Let evil tongues say that we were rivals, that is untrue. True, the esteemed Sir Newton and I worked on the same general question, even before we were acquainted..."

"And simultaneously solved this most important problem!..." Newton chimed in.

"Don't be modest, respected sir," Leibniz interrupted him. "You solved this problem seven years earlier..."

"Yes, but you, dear Mr. Leibniz, discovered the same thing completely independently," Newton

interrupted him in turn.

"Let’s not go into details. Let this discovery belong to both of us," concluded Leibniz.

"Most esteemed Sir Newton and deeply respected Mr. Leibniz," Seva addressed them, "would you be so kind as to tell us what discovery you made?"

"Allow me," said Newton, "to answer your question with another question. Have you been to Mirror Street in the city of Arabella?"

"We have!" blurted out Seva. "We even rode a bus there. And we saw both dwarves and giants."

"Well," continued the English scientist, "those very dwarves and giants were invented by Mr. Leibniz and myself."

"Sir Isaac is quite the joker!" smiled Leibniz. "We didn’t invent any dwarves or giants, but simply introduced the concept of infinitesimally small and infinitely large quantities…"

"And taught people how to use them," Newton finished.

The conversation was unexpectedly interrupted by a plump, rosy-cheeked man in a silk doublet trimmed with the finest lace who approached us.

"I’ll be damned if that isn’t Porthos from 'The Three Musketeers'!" shouted Seva.

"You’ve lost your mind!" Tanya protested. "What would musketeers be doing in Dwarfland?"

"No, no, I’m not a musketeer," laughed the stranger, "although I was personally acquainted with d’Artagnan! I am French too. My name is Pierre Fermat."

"I can assure you," said Newton, "that our dear Fermat is one of the most beloved and respected scientists in Dwarfland."

"Quite understandable," added Leibniz, "because Monsieur Fermat is one of the creators of number theory. If you consider that the Arithmetic state is a country of numbers, you won’t be surprised by Monsieur Fermat’s enormous popularity."

Fermat playfully covered his ears:

"One shouldn’t place the entire weight of glory on one person. Besides me, many others have contributed to number theory. It’s enough to remember the great Greek Pythagoras or all those who lived after me: the St. Petersburg academician Leonhard Euler, the Moscow professor Pafnuty Lvovich Chebyshev, or more recent scientists—Soviet mathematicians Lev Genrikhovich Shnirelman, Ivan Matveyevich Vinogradov… I could list a hundred of them for you. And you attribute everything to me!"

"And yet, dear Monsieur Fermat, what you have done can’t be taken away from you."

Fermat smiled mysteriously and said:

"With all that, I have caused people a lot of trouble."

"It must be pleasant to cause humanity even one such 'trouble'!" remarked Newton.

"None of us would refuse that," added Leibniz.

"What kind of trouble is it?" Seva inquired.

"Monsieur Fermat," answered Newton, "is talking about a theorem he discovered that no one has been able to prove for three hundred years."

"It's called Fermat's Last Theorem!" added Leibniz.

"Why so loud? I never called it 'last,'" objected Fermat. "It came to my mind when I was reading the excellent ancient Greek 'Arithmetic' by Diophantus. A very simple theorem."

"The theorem may be simple," said Leibniz, "and yet no one has found a proof so far!"

"You forget," noted Fermat, "that for some special cases, some have managed to find a proof."

"I'm talking about the fact," replied Leibniz, "that there is still no complete proof of your theorem."

"And did you prove it yourself?" Seva asked Fermat.

"Better not ask, my dear friend," the scientist answered sadly. "I proved it, but... The whole matter is in this little 'but.' I wrote my proof in the margins of Diophantus's book, and, just imagine, that page turned out to be torn out!"

"That's how bad it is to damage books!" Newton remarked with annoyance.

"But you probably remember your proof? Whisper it in my ear, and I'll publish it later," suggested Seva.

The three scientists burst out laughing. They couldn't calm down for a long time.

"My dear Gavroche... I don't know what your real name is," Fermat finally addressed Seva. "Before listening to the proof, one must become familiar with the theorem itself! I'm only afraid that this is somewhat premature. However, if you wish, get a book from the library and familiarize yourself with my theorem. Perhaps someday I will reveal this secret to you."

The scientists ceremoniously said goodbye. And we went forward again.

A man with a proud bearing, wearing a curled powdered wig, was approaching us. He was leading an old man with a small round cap by the arm.

"Mikhail Lomonosov, a pomor from Arkhangelsk province," he introduced himself as he came up to us. "Happy to see compatriots who have visited the great realm of numbers. Numbers bring not only use but also joy to people. Have you advanced much in arithmetic?" Mikhail Vasilyevich asked Seva.

"Not very much," Seva sighed. "Arithmetic is a science oh-oh-oh! Not easily mastered. A difficult science!"

"I suppose you have to acquire knowledge yourself, study books?"

"No," answered Seva, "we study at school, we have a teacher."

"Ah, at school?" Lomonosov was surprised, "And still it’s difficult? Aha, I understand. You want to study, only there’s not enough time. I was also my father’s helper: casting nets into the sea, hauling in the catch. It was also hard for me to study. And there was no money for education."

"No, Mikhail Vasilyevich," objected Seva. "What net? I don’t do anything at home, I don’t even go out for bread. And our education is free."

Lomonosov pondered.

"Strange!" he said. "You don’t work, and you study for free. And it’s still difficult! But I did everything myself. I learned reading, writing, and arithmetic from a book by myself, and only when time remained. 'Arithmetic' by Magnitsky—an excellent book! At nineteen, I set off for Moscow. On foot, in bast shoes, with a bag on my back… That’s how a peasant obtained an education in our time."

"And yet in your time," Tanya interjected, "it was easier to study. After all, you only dealt with arithmetic!"

"Who told you that?!" Lomonosov was indignant. "In my time, young scientists were interested in all sorts of things! Physics, astronomy, chemistry… My friend, the St. Petersburg academician Leonhard Euler, will confirm this."

Lomonosov brought the old man who had been standing aside to us. He smiled, looking somewhere past us with motionless eyes. And we realized that he was blind.

"Yes," said Euler. "We were interested in many things. For example, I studied medicine in my homeland, Switzerland, then physics… And only when I moved to Russia did I firmly dedicate myself to mathematics. Although I was also fascinated by astronomy, even by calculating columns. There is much unexplored in life. You want to know everything, discover everything. And Mikhail Vasilyevich here also wrote poetry."

'"The abyss opened, full of stars, there’s no counting stars, no bottom to the abyss!'" recited Oleg.

"It’s pleasant that you remember that," said Lomonosov. "Who said that science and poetry are different things? In my opinion, to be a good mathematician, one must be a poet at heart. What do you think, Sofya Vasilyevna?" he addressed a young woman passing by.

"You are, as always, right, Mikhail Vasilyevich," she answered. "All my life I was drawn to both mathematics and literature. I wrote scientific treatises, but that didn’t prevent me from composing novels, dramas…"

"Let me introduce you," Lomonosov addressed us. "Sofya Vasilyevna Kovalevskaya, the first Russian woman professor. Tell us, Sofya Vasilyevna, how did your fascination with mathematics begin?"

"It’s very funny," Kovalevskaya blushed, "I’m embarrassed to tell. When I was quite small, for some reason I can’t remember, the walls in my nursery were papered with pages from some book instead of wallpaper. It turned out to be a textbook of higher mathematics. Before my eyes, numbers, unfamiliar signs, formulas always flashed… And seeing them daily, I became so fascinated with them that I decided to dedicate myself to mathematics. So you could say I’m a

wallpaper mathematician!"

"Yes," said Lomonosov, "now Sofya Vasilyevna is joking. But how much suffering she once had to endure! No one recognized a woman's right to learn, let alone to teach others!"

"However," added Sofya Vasilyevna, "men also had it hard. Isn't that right, Nikolai Ivanovich?" she turned to a thin man in a uniform frock coat.

It was the Kazan mathematician Lobachevsky.

"Don't mention it!" Lobachevsky waved his hand. "I don't even want to remember the absurdities that ignorant people spread about my writings. They considered me insane!"

"But now you can triumph," I said. "Your works are world-famous!"

Lobachevsky only smiled modestly.

An old man with a large gray beard approached us. All those standing nearby bowed respectfully to him.

"Pafnuty Lvovich Chebyshev, a native of Kaluga province," he introduced himself.

"Pafnuty Lvovich," I whispered to the children, "at sixteen he was already a university student, and at twenty-five he defended his dissertation."

Chebyshev frowned slightly. He had heard everything and immediately changed the subject.

"How beautifully your dress is sewn!" he addressed Tanya. "I probably wouldn't be able to do it so well."

"You?!" Tanya was surprised. "Are you a tailor?"

"Of course!" laughed Chebyshev. "Cutting clothes is my main profession."

"This shouldn't be taken literally," said a man who approached us after Chebyshev. "Allow me to introduce myself! My name is Alexander Mikhailovich Lyapunov. I know Pafnuty Lvovich very well—he is my teacher. And he taught me not tailoring, but mathematics."

"And who said that a mathematician cannot also be a tailor?" Chebyshev exclaimed heatedly.

"I'll let you in on a secret," smiled Lyapunov. "Professor Chebyshev found a way to cut clothes using mathematical calculations..."

"So am I a tailor or not?" interrupted Chebyshev.

"Of course you're a tailor, if we disregard the fact that you've written dozens of mathematical papers," Lyapunov agreed mischievously. "By the way, did you manage to visit the Alley of Prime Numbers in Dwarfland?" he addressed us. "You did? Excellent! Well, keep in mind that my dear teacher is the most honored guest on that alley. Pafnuty Lvovich did much to facilitate the search for prime numbers. And he was very successful in that!"

"Well, aren't you ashamed!" Chebyshev implored. "You're my student! It appears I taught you to

sing dithyrambs. What will my young compatriots think of me! If they’re interested, they can read my writings themselves."

"Dear Pafnuty Lvovich," exclaimed Lyapunov, "I recognize your professorial absent-mindedness! These nice schoolchildren won’t be able to read a single line of yours. After all, they don’t yet know higher mathematics…"

"So what?" replied Pafnuty Lvovich. "Let them learn. Let them finish school quickly, enter university, and then-then…"

There was nothing to say to that. We said goodbye and moved on again. And the longer we walked, the more scientists we met. There were doctors, physicists, agronomists, writers, biologists, chemists—now not a single science can do without mathematics!

More and more frequently, we heard the roar of airplanes, the chatter of cybernetic machines, the discharges of atomic reactors…

And suddenly we heard a musical phrase—just a few notes. But they were unmistakable. Our call sign!

In an instant, a long glittering arrow shot into the sky, leaving a fiery tail behind it.

And immediately the radio spoke:

"In the Soviet Union, another spaceship has been launched…"

A powerful "hurrah" drowned out the announcer’s voice.

And we didn’t find out what number this latest space rocket was assigned.

A group of people stood on a raised platform. Of course, these were the rocket’s designers.

We pushed forward to see their faces. But we never succeeded, because… because…

We were still sitting in the school garden, at a wooden table.

"What an interesting fairy tale you told," Oleg said thoughtfully.

"Was none of it real?" Seva sighed. "Neither Four with her bow…"

"Nor the ice ballet…" Tanya chimed in.

"Nor Mirror Street…" Oleg continued.

"Maybe it wasn’t," I said. "And maybe it was… Look, here’s a sheet of paper! One of those you just saw gave it to me for you."

I placed on the table a page torn from an ordinary squared notebook. And the children read the words of the great Russian mathematician Nikolai Ivanovich Lobachevsky:

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"It seems impossible to doubt... the truth that everything in the world can be
represented by numbers."
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Golitsyno

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